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ON THE POWER OF THE LOGNORMAL DISTRIBUTION ON SPECIFIED LEVEL OF SIGNIFICANCE

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Abstract

The paper discussed the values and graphs of the power function of hypothesis testing on the lognormal distribution. The research methodology is to derive the formula of the power function and plot curves using R code for 1% level of significance. It is concluded that the degree of freedom, bound of the rejection area, and parameter shape really affect significantly to the curves and values of the power function. The curves of the power are sigmoid, and they increase quickly to be one on the small parameter shape and large degree of freedom.

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1. Introduction

Many authors such as, Pratikno [2], Khan [12-14], Khan and Saleh [15-17, 20, 21], Khan and Hoque [19], Saleh [1], Yunus [6], and Yunus and Khan [7-10], already studied the power and size of the tests on the hypothesis testing. Here, we must choose the maximum power and minimum size. Following, Wackerly et al. [5] and Pratikno [2], the power is the probability to reject H_0 under H_1 in testing hypothesis, and the size is the probability to reject H_0 under H_0, we then write the power and size in testing hypothesis, $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ $(\text{or } H_1 : \theta = \theta_1)$ as, respectively,

$$\pi(\theta_1) = P(\text{reject } H_0 \,|\, \theta = \theta_1) \quad \text{and} \quad \alpha(\theta_0) = P(\text{reject } H_0 \,|\, \theta = \theta_0). \quad (1)$$

Note that equation (1) can be written as $\pi(\theta_1) = 1 - \beta$ and $\alpha(\theta_0) = \alpha$ (tend to be constant), where α is the probability of type error I and β is the probability of type error II. Details of the power and size on several continuous distributions are found in Pratikno et al. [3, 4], and the power and size in testing coefficient parameters on the regression model are found in Pratikno [2].

Here, we note that many authors studied the power in testing intercept with non-sample prior information (NSPI), such as Pratikno [2], Khan and Saleh [20] and Khan [12]. They used the probability integral of the cumulative distribution function (cdf) to compute the power and size. Furthermore, Pratikno [2] and Yunus and Khan [9] used the formula of the power to compute the cdf of the bivariate noncentral F (BNCF) distribution in multivariate and multiple regression models. Moreover, there are some authors who have contributed to the research on the power in the context of the hypothesis testing, such as Khan [12-14], Khan and Saleh [15-17, 20, 21], Khan and Hoque [17], Saleh [1], Yunus [6], and Yunus and Khan [7-10]. In addition, Pratikno [2] and Khan et al. [9] used the

BNCF distribution to compute the power using R code. This is due to the computation of the probability integral of the probability distribution function (pdf) and cdf of the BNCF distribution which are complicated and hard (see Pratikno [2] and Khan et al. [18]).

To investigate the power, we use the research methodology steps as follows: (1) we must determine the sufficiently statistics, (2) we must also create the rejection area using uniformly most powerful test (UMPT), (3) we then derive the formula of the power of the lognormal distribution, and (4) finally we compute and figure the graphs using the generated data (in simulation).

In this paper, Section 1 presents the introduction. The formula and values of the power function and their graphs are given in Section 2. Finally, the conclusion is provided in Section 3.

2. The Values and Graphs of the Power-size of the Lognormal Distribution

2.1. The power of the lognormal distribution

To derive the formula of the power function and figure the graphs, we follow Pratikno [2] by setting the join distribution of the random variable of the lognormal distribution, $X_1, ..., X_n$, as

$$f(x_1, ..., x_2 | \mu) = \prod_{i=1}^{n} f(x_i | \mu) = \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi}\sigma x_i} e^{-\frac{1}{2\sigma^2}[\ln(x_i) - \mu]^2} \right)$$

$$= \frac{1}{\prod_{i=1}^{n} x_i (\sqrt{2\pi}\sigma)^n} e^{\frac{-\sum_{i=1}^{n} (\ln x_i)^2}{2\sigma^2}} e^{\frac{\sum_{i=1}^{n} \ln x_i \mu}{\sigma^2} - \frac{n\mu^2}{2\sigma^2}}. \quad (2)$$

Here, we let $s = \sum_{i=1}^{n} \ln X_i$, and then $s = \sum_{i=1}^{n} \ln X_i \sim N(n\mu, n\sigma^2)$ will be sufficient statistics. Furthermore, we used UMP test in testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu > \mu_0$, to find the rejection area in rejecting H_0, which occurred on $\sum_{i=1}^{n} \ln x_i \geq Z_\alpha \sigma \sqrt{n} + n\mu_0$. Therefore, we obtain the power function as

$$\pi(\mu) = P(\text{reject } H_0 \mid \mu) = P\left(\sum_{i=1}^{n} \ln x_i \geq Z_\alpha \sigma \sqrt{n} + n\mu_0 \mid \mu\right). \qquad (3)$$

Using equation (3), we produce the graphs of the power function of the lognormal distribution in Figure 1.

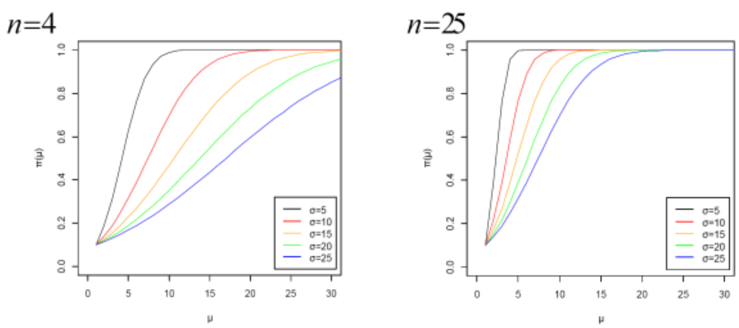

Figure 1. The power curves of the lognormal distribution for $n = 4$ and 25 and σ at $\alpha = 0.01$.

From Figure 1, we see that the curves decrease as σ increases, but they are going to one (quickly) for large *n*. It means that both *n* and σ are really significant effect to the skewness of their curves. Due to the size being constant, we simulate the values of the size on different $\alpha = 0.01$ and 0.05, the results showed that the value of the size is 0.049 when $\alpha = 0.05$ and the value of the size is 0.10 for $\alpha = 0.01$. The graphs of the size are then presented in Figure 2. We therefore choose the small size 0.04998 on $\alpha = 0.05$. The detail of the computation of the values of the size on different levels of significance is given as: (1) for $n = 25$, $\alpha = 0.01$, the size (α), $\alpha = P\left(\sum_{i=1}^{n} \ln x_i \geq 2.327\sigma\right) = 1 - \Phi(2.327) = 0.01001$, and (2) for $n = 25$, $\alpha = 0.05$, $\alpha = P\left(\sum_{i=1}^{n} \ln x_i \geq 1.645\sigma\right) = 1 - \Phi(1.645) = 0.04998$.

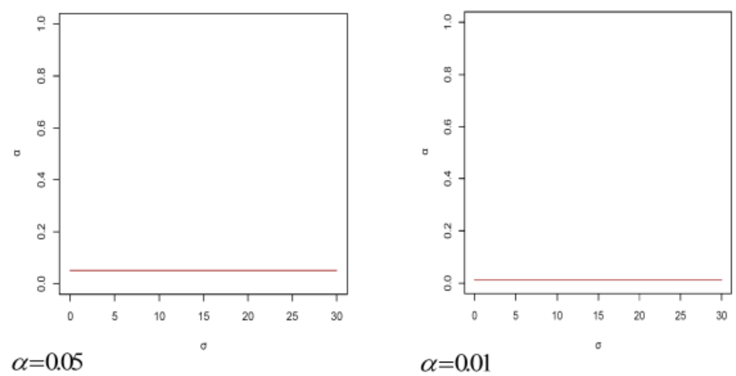

Figure 2. The graphs of the size on $\alpha = 0.05$ and 0.01.

2.2. The values of the power of the lognormal distribution

In this subsection, we present the details of the values of the power function of the lognormal distribution for several σ on $n = 4$ and 25 in Tables 1 and 2.

Table 1. The values of the power on $n = 4$, $\alpha = 0.01$

μ	$\pi(\mu)$				
	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$
1	0.0100	0.0100	0.0100	0.0100	0.0100
2	0.0270	0.0167	0.0142	0.0130	0.0123
3	0.0635	0.0270	0.0197	0.0167	0.0151
4	0.1301	0.0422	0.0270	0.0214	0.0185
5	0.2339	0.0635	0.0365	0.0270	0.0224
6	0.3722	0.0924	0.0485	0.0339	0.0270
7	0.5294	0.1300	0.0635	0.0421	0.0324
8	0.6822	0.1772	0.0818	0.0519	0.0386
9	0.8089	0.2339	0.1039	0.0635	0.0458
10	0.8986	0.2994	0.1300	0.0769	0.0541
11	0.9529	0.3722	0.1604	0.0924	0.0635
12	0.9809	0.4498	0.1950	0.1100	0.0740
13	0.9933	0.5294	0.2339	0.1300	0.0859
14	0.9979	0.6079	0.2767	0.1524	0.0992
15	0.9994	0.6822	0.3229	0.1772	0.1139
16	0.9998	0.7498	0.3722	0.2044	0.1300
17	0.9999	0.8089	0.4236	0.2339	0.1477
18	0.9999	0.8585	0.4763	0.2656	0.1670
19	0.9999	0.8986	0.5294	0.2994	0.1878
20	1.000000	0.9297	0.5821	0.3350	0.2101

Table 2. The values of the power on $n = 25$, $\alpha = 0.01$

μ	$\pi(\mu)$				
	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$
1	0.0100	0.0100	0.0100	0.0100	0.0100
2	0.0924	0.0339	0.0231	0.0189	0.0167
3	0.3722	0.0924	0.0485	0.0339	0.0270
4	0.7498	0.2044	0.0924	0.0575	0.0421
5	0.9529	0.3722	0.1604	0.0924	0.0635
6	0.9962	0.5690	0.2548	0.1409	0.0924
7	0.9998	0.7498	0.3722	0.2044	0.1300
8	0.9999	0.8798	0.5029	0.2823	0.1772
9	1.0000	0.9529	0.6333	0.3722	0.2339
10	1.0000	0.9851	0.7498	0.4697	0.2994

From Tables 1 and 2, it is clear that the value of the power function quickly approaches one on small σ and large *n*. It means that both *n* and σ really affect to sigmoid and skewness of the curves. We then note that the values of the power function in Figure 1 and Figure 2 always increase to become one for both *n* and σ.

Based on the values of the power listed in Tables 1 and 2, we present several graphs of the power on different levels of significance (0.05 and 0.10), see Figure 3.

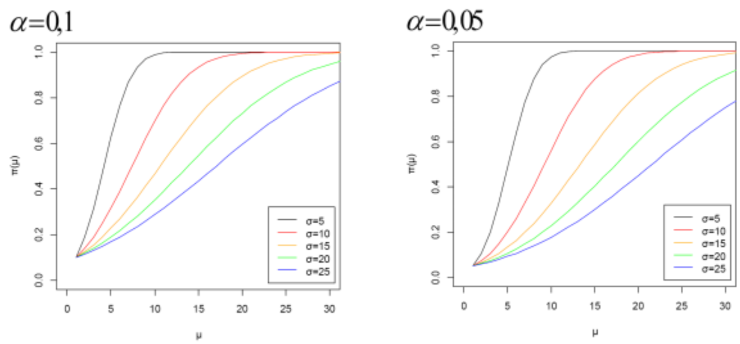

Figure 3. The graphs of the power at levels of significance 0.10 and 0.05.

From Figure 3, it is clear that the power quickly approaches one when the standard deviation decreases (or becomes small) for both the levels of significance. We therefore conclude that all the powers become one quickly for small standard deviation on levels of significance 0.01, 0.05 and 0.10.

3. Conclusion

There are several steps in the derivation of the power of the lognormal distribution. One of the important things is to find the rejection area using UMPT test. The results show that the curves decrease as σ increases, and they quickly reach one for large *n* and small σ.

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