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The Power of Weibull and Exponential Distributions On Testing Parameters Shape

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Abstract. We study the power in testing parameter shape of the Weibull and Exponential distributions and analyze it graphically. The power and plot of their graphs are computed using *R*-code. The results showed that the power of the distribution is depended on the parameter shapes.

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*Keyword***:** Distributions, parameter shape and power and size.

1. Introduction

The concept of power is defined as probability to reject H_0 *under* $H_a:\theta=\theta_a$ for testing hypothesis $H_0:\theta=\theta_0$ versus $H_1:\theta\neq\theta_0$, on parameter θ. The size is then given *under* $H_0:\theta=\theta_0$. Here, we then wrote as $\pi(\theta_a)=P(reject\ H_0|\theta=\theta_a)$ and $\alpha^*=\alpha(\theta_0)=P(reject\ H_0|\theta=\theta_0)$ (Wackerly [4]). Note that α (level of significant) is commonly a special case of the $\alpha^* = \alpha(\theta)$.

Many authors already studied the power and size in computing the probability integral of the cumulative distribution function (cdf) of the distributions in testing intercept using non-sample prior information (NSPI), such as Pratikno [2], Khan and Pratikno [7] and Khan [8]. Moreover, Pratikno [3] and Khan et al. [14] already used the power ans size to compute the cdf of the bivariate noncentral *F (*BNCF*)* distribution of the pre-test test (PTT) in multivariate simple regression model (MSRM), multiple regression model (MRM) and parallel regression model (PRM). Furthermore, Khan [8,9], Khan and Saleh [11, 12,13], Khan and Hoque [10], Saleh [1], Yunus [6], and Yunus and Khan [5] also contributed in computing the values of the power of the test (PTT) on the estimation areas. In the context of the hypothesis testing with NSPI, the bivariate noncentral *F* distribution is used to compute the power of the pre-test test (PTT) on the MSRM, MRM and PRM. The formula of the power and size of the tests of the UT, RT and PTT are found in Pratikno [3] in testing hypothesis one-side or two-side hypothesis. Due to the probability integral of the power and size of the PTT is not simple and tend to be complex, so they are computed using *R-code*. The detail of the BNCF is found on Pratikno [2] and Khan et al.[14].

To compute the power of the Weibull and Exponential distributions and its application on the regression models, the steps of the research methodology are (1) find the sufficiently sstatistics, (2) determine the rejection area of the distributions using *uniformly most powerful test* (UMPT), and (3) derive the formula of the power of the distributions in testing one-side (or two-side) hypothesis.

The research presented the introduction in Section 1. Analysis of the power and size of the distributions are obtained in Section 2. Section 3 described the conclusion of the research.

2. The Power of the Distributions

2.1.The Power of the Weibull Distribution

In this section, we presented the formula and graphs of the power in testing parameters shape (δ , β) for one-side hypothesis on the Weibull distribution. The procedures are as follow: (1) find the statistics cukup, (2) determine the rejection area of the Weibull distribution using *uniformly most powerful test* (UMPT), (3) derive the formula of the power and compute the values of power and then plot them. This distribution is often applied in life testing of the components, so it is like Exponential distribution.

Let, X be a random variable follows the Weibull distribution, the cdf and probability density function (pdf) of this distribution are then given as, respectively,

$$F(x)=\begin{cases}1-e^{-\left(\frac{x}{\delta}\right)^{\beta}} & ,x\geq 0\\ 0 & ,\text{otherwise}\end{cases} \quad (1)$$

with parameter shape $\delta > 0$ and scale parameter $\beta > 0$, and

$$f(x)=\frac{dF(x)}{dx}=f(x)=\begin{cases}\frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1}e^{-\left(\frac{x}{\delta}\right)^{\beta}} & ,x\geq 0\\ 0 & ,\text{otherwise}\end{cases} \quad (2)$$

To compute the power of the distribution, we have to compute the sufficiently sstatistics. It is used to find the rejection area, as follow: (1) first define the *likelihood* function of the Weibull distribution as

$$f(x_1,\ldots,x_n \mid \delta)=g(s,\delta)\cdot h(x_1,\ldots,x_n) \quad (3)$$

with $f(x)=\frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1}e^{-\left(\frac{x}{\delta}\right)^{\beta}}$, $f(x_1,\ldots,x_n \mid \delta)=\prod f(x_i \mid \delta)=\frac{\beta}{\delta}\left(\frac{1}{\delta}\right)^{\beta-1}\left(\prod_{i=1}^{n}x_i\right)^{\beta-1}e^{-\left(\frac{1}{\delta}\right)^{\beta}\left(\sum_{i=1}^{n}x_i^{\beta}\right)}$,

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$$g(s,\delta)=\frac{\beta}{\delta}\left(\frac{1}{\delta}\right)^{\beta-1} e^{\left(\frac{1}{\delta}\right)^{\beta} s}, \quad h(x_i)=\prod_{i=1}^{n}(x_i)^{\beta-1}, i=1,2,\ldots,n, \text{ and } s=\sum_{i=1}^{n} x_i^{\beta} .$$

(2) using mathematical technique, we then get, $s=\sum_{i=1}^{n} x_i^{\beta}$ be sufficiently statistics of the parameter δ of the Weibull distribution, (3) the rejection region (RR) is found by UMPT and we then got as $P(s>\chi^2_{(2n,\alpha)})$, with s is sufficient statistics and δ is parameter shape of the Weibull distribution, and (4) finally, we derive the formula of power of the Weibull distribution for one-side testing hypothesis, $H_0:\delta = \delta_0$ versus $H_1:\delta>\delta_1$, is given as

$$\pi(\delta)=P(\text{reject } H_0 \mid \text{un der } H_1)=P\left(\sum_{i=1} x_i^{\beta}>k\right)=P\left(\frac{2}{\delta^{\beta}}\sum_{i=1}^{n} x_i^{\beta}>c\right)$$
$$=P\left(\sum_{i=1}^{n} x_i^{\beta}>\chi^2_{(2n,\alpha)}\frac{\delta_0^{\beta}}{2}\right)=P\left(\chi^2>\left(\frac{\delta_0}{\delta}\right)^{\beta}\chi^2_{(2n,\alpha)}\right), \text{ with } c=\chi^2_{(2n,\alpha)}. \quad (4)$$

Following Pratikno [3] (here, $\alpha=0.1$, $n=10$ *and* 30) and using the equation (4), we then get the graphs of the power for $\alpha=0.05$and n=40 , β =2,3,4,5, on hypothesis testing $H_0:\delta=\delta_0=1$ versus $H_1:\delta_0>1$, are presented in Figure 1.

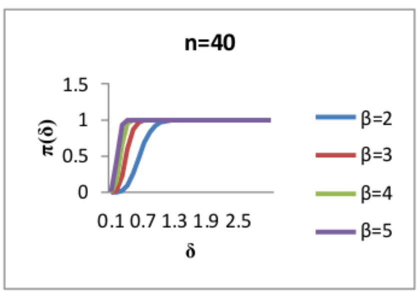

Figure 1. The graphs of power in testing parameter δ at $\alpha=0.05$

Figure 1. showed that the graphs of the power tend to increase as the sample size (n) and β increase. In our simulation, we see that α has a little significant influence to the curve of the power of the parameter shape, especially when n = *30* (see Pratikno [3])

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2.2.The Power of the Exponential Distribution

Similarly (see Section 2.1.), we then derived the graphs of the power in testing parameters shape (θ) for one-side hypothesis, $H_0:\theta=\theta_0$ versus $H_1:\theta>\theta_0$, on the Exponential distribution. Let, X be a random variable follows the Exponential distribution, the probability density function (pdf) is given as $f(\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}}, x>0$

sehingga $f(x_1,\cdots x_n|\theta)=\prod_{i=1}^{n} f(x_i|\theta)=\frac{1}{\theta^n}e^{-\frac{1}{\theta}\left(\sum_{i=1}^{n}x_i\right)}$. Therefore, we got $g(s,\theta)=\frac{1}{\theta^n}e^{-\frac{1}{\theta}(s)}$ with $h(x_i)=1$ nd sufficiently statistics s= $\left(\sum_{i=1} x_i\right)$.

By definition of the power and size, we derive the formula of the power and size of the Exponential distribution in testing parameter shape of the hypothesis, $H_0:\theta=\theta_0$ versus $H_1:\theta>\theta_0$, as follow, respectively.

$$\begin{aligned}\pi(\theta) &= P(reject\ H_0|under\ H_1)=P\left(\sum_{i=}^{n} x>k\right)=P\left(\frac{2}{\theta^n}\sum_{i=}^{n} x>\chi^2_{2n}\right)\\ &=P\left(\chi^2>\frac{\theta_1}{\theta^n}\chi^2_{2n}\right)=P\left(\chi^2>c_1\right)=\int_{c_1} f(x)dx, \text{ and}\end{aligned} \quad (5)$$

$$\begin{aligned}\alpha(\theta_0) &= \alpha(\theta)=P(reject\ H_0|under\ H_0)=P\left(\chi^2>\frac{\theta_0}{\theta^n}\chi^2_{2n}\right)\\ &=P\left(\chi^2>c_0\right)=\int_{c_0}^{\infty} f(x)dx,\end{aligned} \quad (6)$$

where *f(x)* follows Chi-Square distribution with *2n* degrees of freedom. Due to the probability integral of the power and size in the equation (5) and (6) are not simple and very complex, so they are computed using *R-code*. Similarly, the graphs are also figured using R-code. From the equation (5) and (6), we see that the power and size are influenced by parameter shape as well.

3. Conclusion

The reserach studied the power in testing parameter shape of the distributions and analyze it graphically. To compute the power and plot of their graphs, *R-code* is used. The results showed that the power of the distribution is influenced by the parameter shapes.

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