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Preface

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PREFACE

The International Conference on Applied Sciences Mathematics and Informatics (ICASMI) is a biennial event hosted by University of Lampung that brings together academics, scholars and researchers from around the world to meet and exchange the latest ideas, networking, opening collaboration research and discuss issues concerning all fields of sciences, mathematics, informatics and their application. It also allows representatives of industry, government employers and postgraduate students to have an opportunity to discuss with experts on some issues they concern. Due to the COVID-19 pandemic, this time the conference was held virtually.

This conference was held from 3rd to 4th of September 2020, in the Faculty of Mathematics and Natural Sciences, Universitas Lampung, Bandar Lampung, Indonesia. Zoom Meeting was utilized as a means of the conference. Each keynote speaker was given 30-minutes for his/her presentation with 15 minutes discussion, while for the oral presentation was held in a parallel session of three or four speakers where each participant was given 10 minutes for presentation and 15 minutes for panel discussion. The participants came from across several institutions and universities from 4 countries. Our initial target participants were 150, fortunately, on the closing date of registration, there were 178 participants who registered from 5 main fields of natural sciences. The main drawback of such virtual conference was the internet connection. A few numbers of speakers had this problem, so they were unable to give their best presentation, however, this drawback did not affect much the quality of this conference.

The theme of this year's conference is "Natural Sciences, Mathematics and Informatics in the Industrial Revolution (IR) 4.0 toward the Sustainable Development Goals (SDGs)." The conference will provide researchers and scientists from mathematics and computer science, researchers from various application areas such as physics, chemistry, life sciences, and engineering, as well as in education and social fields, to discuss problems and solutions in the area, to identify new issues, and to shape future directions for research.

We would like to acknowledge all of those who have supported the 3rd ICASMI. Each individual and institutional help were very important for the success of this conference. We would like to thank the keynote speakers who are competent in their field of study and come from different countries, such as, Japan, Malaysia, Turkey and Indonesia, and the organizing committee for their valuable advice in the organization and helpful peer review of the papers.

We hope that this conference would be a forum for excellent discussions that put forward new ideas and promote collaborative research. We are sure that the IOP proceedings publication will serve as an important research source of references and the knowledge, which will lead to not only scientific and engineering progress but also other new products and processes.

Chair,

Prof. Dr. Rudy Situmeang

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Committee of the 3rd International Conference on Applied Sciences Mathematics and Informatics (3rd ICASMI) 2020

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Photos

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Figure 1 Keynote Speaker by Kenji Satou

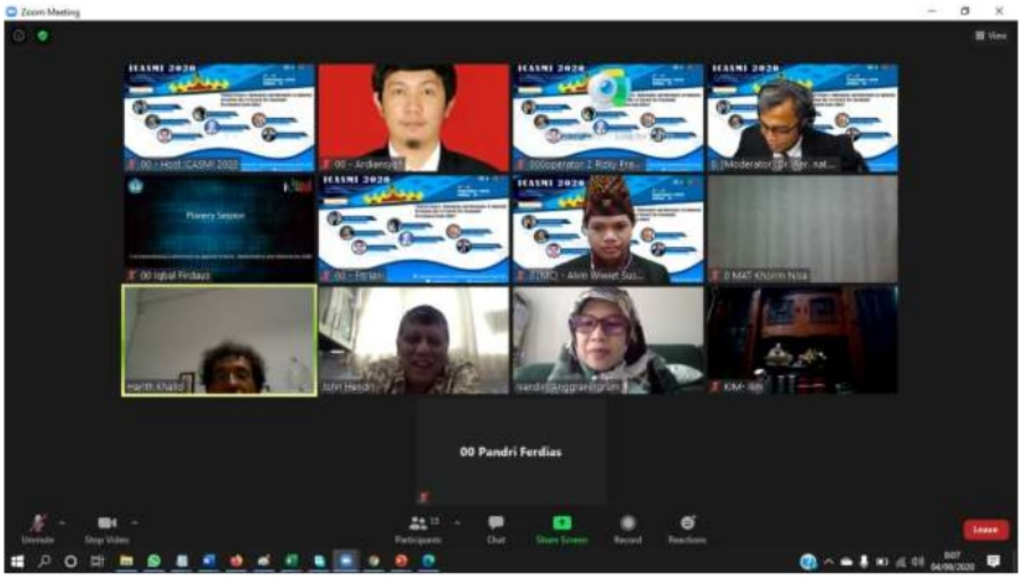

Figure 2 Keynote Speaker by Prof Harith, Prof John Hendri, and Prof Ivandini

Figure 3 Openning Speach by Chief of Committee

Figure 4 Speech by the Dean of Faculty of Mathematics and Natural Sciences, UNILA

Figure 5 Committee

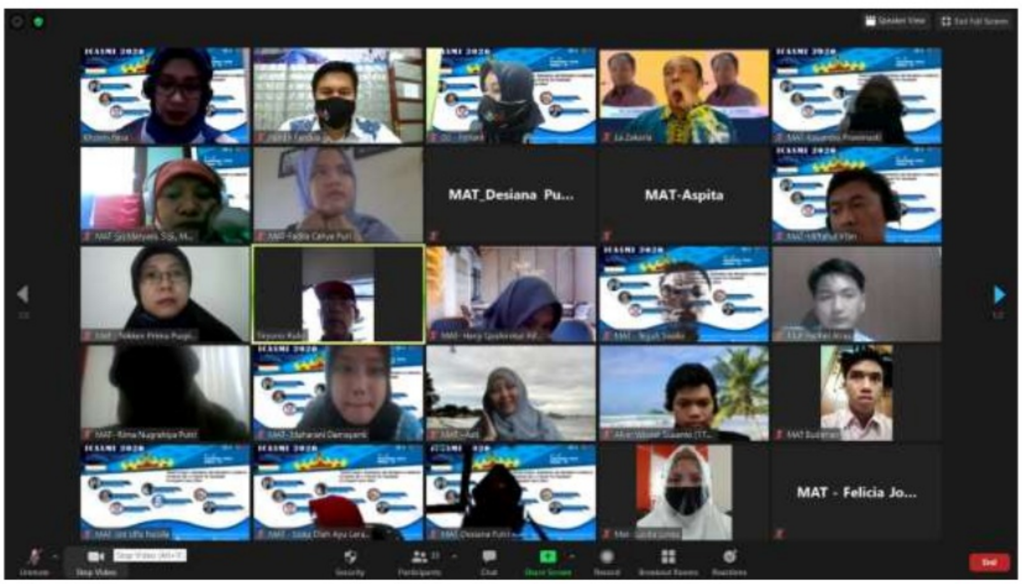

Figure 6 Parallel Session

Figure 7 Closing Speech by Heri Satria

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Peer review declaration

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Peer review declaration

All papers published in this volume of Journal of Physics: Conference Series have been peer reviewed through processes administered by the Editors. Reviews were conducted by expert referees to the professional and scientific standards expected of a proceedings journal published by IOP Publishing.

- **Type of peer review: Double-blind**

  Conference submission management system: The submission process used was Google form

  Number of submissions received: We received 171 papers submitted plus 7 keynote presentation but the keynote speakers were not ready on the due date to submit their full paper, so their papers were not reviewed

  - **Number of submissions sent for review: 152**
  - **Number of submissions accepted: 112**

  Acceptance Rate (Number of Submissions Accepted / Number of Submissions Received X 100): The acceptance rate was: (112/152) x 100% = 73.68%

  - **Average number of reviews per paper: 3.38**
  - **Total number of reviewers involved: 45**

  Any additional info on review process: The review process was carried using single-blind review process was to minimize the expenses we had to spent as the ICASMI committee, although not much, gave certain amount of payment for per article reviewed by reviewers, thus if we used double-blind review, it will cost double.

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Solution Formula of the Compressible Fluid Motion in Three Dimension Euclidean Space using Fourier Transform

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Solution Formula of the Compressible Fluid Motion in Three Dimension Euclidean Space using Fourier Transform

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Abstract. We derive a detailed determination of the solution formula for the compressible viscous fluid flow in three dimensional Euclidean space using Fourier transform. For the further research, we can not only generalized the model problem to the N-dimensional Euclidean space (N>3) but also we can estimate the solution operator families of the model problem.

Keywords: compressible, viscous fluid, Euclidean space, Fourier transform.

1. Introduction

In this paper, we consider the solution formula of the linearized for compressible fluid flow which described as follows:

$$\begin{cases} \rho_t + \gamma \mathrm{div}\, \mathbf{v} = 0 \\ \mathbf{v_t} - \mu\Delta\mathbf{v} - \nu\nabla\mathrm{div}\,\mathbf{v} + \gamma\nabla\rho = 0. \end{cases}, \qquad (1)$$

with the initial data are $v|_{\partial\Omega} = 0$, $(\rho, \mathbf{v})|_{t=0} = (\rho_0, v_0)$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle^T$ is velocity and $\langle v_1, v_2, v_3 \rangle^T$ is the transpose of $\langle v_1, v_2, v_3 \rangle$. We know that density and velocity are written as $\rho = \rho(x)$ and $\mathbf{v} = \langle v_1(x), v_2(x), v_3(x) \rangle^T$, respectively. Moreover, μ and γ are positive constants, and ν is a constant such that $\mu + \nu > 0$ and μ and ν are the viscosity coefficients. The domain Ω is a three dimensional Euclidean space $\mathbb{R}^3$. This result can be generalized to $\mathbb{N}$-dimensional case and also we can estimates the solution operator families of the model problem which investigated by [2] in 2016.

To introduce our main result, first of all we introduce the notation. For a scalar-valued function $u = u(x)$ and vector-valued function $\mathbf{v} = \mathbf{v}(x) = \langle v_1(x), v_2(x), v_3(x)\rangle^T$, we set for $\partial_k = \frac{\partial}{\partial x_k}$, $(k = 1, \cdots, N)$

$$\nabla u = (\partial_1 u, \partial_2 u, \partial_3 u)^T, \quad \Delta u = \sum_{k=1}^{3} \partial_k^2 u, \quad \Delta\mathbf{v} = \langle \Delta v_1, \Delta v_2, \Delta v_3\rangle^T,$$

$$\mathrm{div}\,\mathbf{v} = \sum_{k=1}^{3} \partial_k v_k, \quad \nabla v = \{\partial_k v_\ell \mid k, \ell = 1,2,3\}, \quad \nabla^2\mathbf{v} = \{\partial_k \partial_\ell v_m \mid k, \ell, m = 1, 2, 3\}.$$

The set of all natural number is denoted by $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We set,

$$W_p^{k,m}(\Omega) = \{ \mathbb{U} = (\rho, \mathbf{v}) \mid \rho \in W_p^k(\Omega), \mathbf{v} \in W_p^m(\Omega) \}.$$

Before we state the main result, first of all we introduce the definition of Sobolev space $W_q^m(\Omega)$.

Definition 1.1 (Adams and Fournier, [1])

Let $k \in \mathbb{N} \cup \{0\}$ and $p \in [1, \infty)$ then the Sobolev Space $W_q^m(\Omega)$ is defined by

$$W_q^m(\Omega) := \{\mathbf{u} \in L_q(\Omega) \mid D^\alpha \mathbf{u} \in L_q(\Omega), \forall\, \alpha \text{ with } |\alpha| \leq m\}$$

Next, we state the main result of this paper.

Theorem 1.2 Let $\lambda_j(\xi), j = 1, \ldots, 4$ be the roots of $\det[\lambda\mathbb{I} + \widehat{\mathbb{M}}(\lambda)] = 0$, where $\lambda_3(\xi) = \lambda_4(\xi) = -\mu|\xi|^2$. Then for $\lambda_j(\xi), j = 1,2$, we have the following assertions:

i. For $|\xi| \geq \frac{2\gamma}{(\mu+\nu)}$, we have $\lambda_j(\xi), j = 1,2$ as follows:

$$\lambda_1 = \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}$$
$$\lambda_2 = \frac{-(\mu+\nu)}{2}|\xi|^2 - \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}.$$

ii. For $|\xi| \leq \frac{2\gamma}{(\mu+\nu)}$, we have $\lambda_j(\xi), j = 1,2$ as follows:

$$\lambda_1 = \overline{\lambda_2}\ \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{i}{2}|\xi|\sqrt{4\gamma^2 - (\mu+\nu)^2|\xi|^2},\ \ i = \sqrt{-1}.$$

iii. For $|\xi| \neq \frac{2\gamma}{(\alpha+\beta)}$, we have the solution formula of $\hat{\rho}(\xi,t)$ and $\hat{\mathbf{v}}(\xi,t)$ as follow

$$\hat{\rho}(\xi,t) = \left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)t} - \lambda_1(\xi)e^{\lambda_2(\xi)t}}{\lambda_2(\xi) - \lambda_1(\xi)}\right)\hat{\rho}_0(\xi,t) - i\left(\frac{e^{\lambda_2(\xi)t} - e^{\lambda_1(\xi)t}}{\lambda_2(\xi) - \lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi),$$

$$\hat{\mathbf{v}}(\xi,t) = i\gamma\xi\left(\frac{e^{\lambda_2(\xi)t} - e^{\lambda_1(\xi)t}}{\lambda_2(\xi) - \lambda_1(\xi)}\right)\hat{\rho}_0(\xi,t) + e^{-\mu|\xi|^2 t}\hat{\mathbf{v}}_0(\xi)$$
$$+\left(\frac{\lambda_2(\xi)e^{\lambda_2(\xi)t} - \lambda_1(\xi)e^{\lambda_1(\xi)t}}{\lambda_2(\xi) - \lambda_1(\xi)} - e^{-\alpha|\xi|^2 t}\right)\frac{\xi\xi^T\hat{\mathbf{v}}_0(\xi)}{|\xi|^2}.$$

This paper is organized as follows: the next section introduce a reduced system for (1) and shows that **Theorem 1.2** follows from the main result for the reduced system. In third Section, we calculate representation formulas for solutions of the reduced system by using the Fourier transform with respect to $\mathbf{x} = \langle x_1, x_2, x_3\rangle$ and its inverse transform. Section 4 proves our main theorem for the reduced system by results obtained in Section 3.

2. Reduced System

In this section, we consider the resolvent problem of eqaution system (1). Set $u_j = v_j$ $(j = 1,2,3)$, then $\mathbf{v} = \langle v_1, v_2, v_3\rangle^T$ satisfies

$$\begin{cases} \lambda\rho + \gamma \operatorname{div} \mathbf{v} = \tilde{f} & \text{in } \Omega, \\ \lambda\,\mathbf{v} - \mu\Delta\mathbf{v} - \nu\nabla\operatorname{div}\mathbf{v} + \gamma\nabla\rho = \tilde{\mathbf{g}} & \text{in } \Omega. \end{cases} \tag{2}$$

First of all, we can write the equation system of (2) in the following

$$(\lambda + \mathbb{M})\mathbb{U} = \mathbb{F} \qquad in\ \Omega \tag{3}$$

where,

$$\mathbb{U} = \begin{pmatrix}\rho \\ \boldsymbol{v}\end{pmatrix} = \begin{pmatrix}\rho \\ v_1 \\ v_2 \\ v_3\end{pmatrix}, \quad \mathbb{M} = \begin{bmatrix} 0 & \gamma div \\ \gamma\nabla & -\mu\Delta - \nu\nabla div\end{bmatrix}, \ and \quad \mathbb{F} = \begin{pmatrix}\tilde{f} \\ \tilde{\boldsymbol{g}}\end{pmatrix}.$$

We can also write the equation system of (2) in a matrix form

$$\lambda\begin{pmatrix}\rho \\ \boldsymbol{v}\end{pmatrix} + \begin{bmatrix} 0 & \gamma div \\ \gamma\nabla & -\mu\Delta - \nu\nabla div\end{bmatrix}\begin{pmatrix}\rho \\ \boldsymbol{v}\end{pmatrix} = \begin{pmatrix}\tilde{f} \\ \tilde{\boldsymbol{g}}\end{pmatrix},$$

or we can write in the form

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$$\lambda\begin{pmatrix}\rho\\ \boldsymbol{v}\end{pmatrix}+\begin{bmatrix}0 & \gamma\sum_{k=1}^{3}\partial_k\\ \gamma\nabla & -\mu\partial_k^2-\nu\partial_k\sum_{j=1}^{3}\partial_j\end{bmatrix}\begin{pmatrix}\rho\\ \boldsymbol{v}\end{pmatrix}=\begin{pmatrix}\tilde{f}\\ \tilde{\boldsymbol{g}}\end{pmatrix}. \quad (4)$$

In addition, for the time derivative, we can write the equation system of (1) in the following

$$\begin{cases}\rho_t+\gamma div\,\boldsymbol{v}=\;0 & in\;[0,\infty)\times\mathbb{R}^3,\\ v_t-\mu\Delta\boldsymbol{v}-\nu\nabla div\,\boldsymbol{v}+\gamma\nabla\rho=0 & in\;[0,\infty)\times\mathbb{R}^3.\end{cases} \quad (5)$$

Furthermore, we consider the equation (5). For the simplicity, we can write the equation (5) to be

$$\mathbb{U}_t+\mathbb{M}\mathbb{U}=0\quad in\;[0,\infty)\times\mathbb{R}^3,\;\;\mathbb{U}\;|_{t=0}=\mathbb{U}_0\quad in\;\mathbb{R}^3, \quad (6)$$

with domain $D(\mathbb{M})=\{\mathbb{U}=(\rho,\boldsymbol{v})\in W_p^{1,2}\mid \boldsymbol{v}\;|_{\partial\Omega}=0\}$, $\mathbb{U}_0=(\rho_0,\boldsymbol{v}_0)$. Then, by taking Fourier transform to (6) with respect to the x variable and solving the ordinary differential equation with respect to variable t, we have

$$\mathbb{U}_t=\mathbb{E}(t)\mathbb{F}=\mathcal{F}^{-1}\left(e^{-t\widehat{\mathbb{M}}(\xi)}\widehat{\mathbb{F}}(\xi)\right),$$

where we define the Fourier transform $\hat{f}$of $f=f(x)$ with respect to $\boldsymbol{x}=\langle x_1,x_2,x_3\rangle$ and its inverse transform as follows:

$$\hat{f}\;=\mathcal{F}_x[f]\,(\xi)=\hat{u}\,(x)=\int_{\mathbb{R}^3}e^{-ix\cdot\xi}\,f(x)dx$$

$$\mathcal{F}_\xi^{-1}[g](x)=\frac{1}{(2\pi)^3}\int_{\mathbb{R}^3}e^{ix\cdot\xi}\,g(\xi)d\xi$$

where $\xi=\langle\xi_1,\xi_2,\xi_3\rangle\in\mathbb{R}^3$.

Next, we consider the resolvent problem of equation system (5) then applying Fourier transform, we have

$$\lambda\widehat{\mathbb{U}}+\widehat{\mathbb{M}}(\xi)\widehat{\mathbb{U}}=\widehat{\mathbb{F}}$$

$$[\lambda+\widehat{\mathbb{M}}(\xi)]\widehat{\mathbb{U}}=\widehat{\mathbb{F}}$$
$$\widehat{\mathbb{U}}=[\lambda I+\widehat{\mathbb{M}}(\xi)]^{-1}\widehat{\mathbb{F}}(\xi)$$
$$\mathbb{U}=\mathcal{F}^{-1}\left\{[\lambda I+\widehat{\mathbb{M}}(\xi)]^{-1}\widehat{\mathbb{F}}(\xi)\right\} \quad (7)$$

with $\det[\lambda I+\widehat{\mathbb{M}}(\xi)]\neq 0$ and $[\lambda I+\widehat{\mathbb{M}}(\xi)]^{-1}$ is inverse of $[\lambda I+\widehat{\mathbb{M}}(\xi)]$.
Furthermore, by using adjoint of the matrix, we can determine the inverse of matrix $[\lambda I+\widehat{\mathbb{M}}(\xi)]$,

$$[\lambda I+\widehat{\mathbb{M}}(\xi)]^{-1}=\frac{1}{\det[\lambda I+\widehat{\mathbb{M}}(\xi)]}\operatorname{adj}[\lambda I+\widehat{\mathbb{M}}(\xi)]. \quad (8)$$

3. Representation formulas for solution

In this section, following [4, section III], we compute representation formulas for solutions of (2). First of all, applying the Fourier transform to matrix $\mathbb{M}$ in equation (3) yield the following matrix

$$\widehat{\mathbb{M}}(\boldsymbol{\xi})=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{i\gamma\xi_1} & \boldsymbol{i\gamma\xi_2} & \boldsymbol{i\gamma\xi_3}\\ \boldsymbol{i\gamma\xi_1} & \boldsymbol{\mu|\xi|^2+\nu\xi_1^2} & \boldsymbol{\nu\xi_1\xi_2} & \boldsymbol{\nu\xi_1\xi_3}\\ \boldsymbol{i\gamma\xi_2} & \boldsymbol{\nu\xi_2\xi_1} & \boldsymbol{\mu|\xi|^2+\nu\xi_2^2} & \boldsymbol{\nu\xi_2\xi_3}\\ \boldsymbol{i\gamma\xi_3} & \boldsymbol{\nu\xi_3\xi_1} & \boldsymbol{\nu\xi_3\xi_2} & \boldsymbol{\mu|\xi|^2+\nu\xi_3^2}\end{bmatrix}$$

where $|\boldsymbol{\xi}|^2=\xi_1^2+\xi_2^2+\xi_3^2$ and $\boldsymbol{i}^2=-\boldsymbol{1}$. Then we also have

$$[\lambda I + \widehat{\mathbb{M}}(\xi)] = \begin{bmatrix} \lambda & i\gamma\xi_1 & i\gamma\xi_2 & i\gamma\xi_3 \\ i\gamma\xi_1 & \lambda + \mu|\xi|^2 + \nu\xi_1^2 & \nu\xi_1\xi_2 & \nu\xi_1\xi_3 \\ i\gamma\xi_2 & \nu\xi_2\xi_1 & \lambda + \mu|\xi|^2 + \nu\xi_2^2 & \nu\xi_2\xi_3 \\ i\gamma\xi_3 & \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \lambda + \mu|\xi|^2 + \nu\xi_3^2 \end{bmatrix}.$$

Then, we calculate for the determinant of matrix $[\lambda I + \widehat{\mathbb{M}}(\xi)]$ using expansion by cofactors, that is

$$\begin{aligned}\det[\lambda I + \widehat{\mathbb{M}}(\xi)] &= \hat{a}_{11}\hat{C}_{11} + \hat{a}_{12}\hat{C}_{12} + \hat{a}_{13}\hat{C}_{13} + \hat{a}_{14}\hat{C}_{14} \\ &= \lambda|\widehat{\mathbb{A}}_{11}| - i\gamma\xi_1|\widehat{\mathbb{A}}_{12}| + i\gamma\xi_2|\widehat{\mathbb{A}}_{13}| - i\gamma\xi_3|\widehat{\mathbb{A}}_{14}| \end{aligned} \quad (9)$$

where $|\widehat{\mathbb{A}}_{ij}|$ is determinant of submatrix that remains after the i-th row and j-th column are deleted from matrix $[\lambda I + \widehat{\mathbb{M}}(\xi)]$ and the number $(-1)^{i+j}\widehat{\mathbb{A}}_{ij}$ is denoted by $\hat{C}_{ij}$ and called the cofactor.
In fact, for $i = j = 1$ the component of $|\widehat{\mathbb{A}}_{ij}|$, we have

$$|\widehat{\mathbb{A}}_{11}| = \begin{vmatrix} \lambda + \mu|\xi|^2 + \nu\xi_1^2 & \nu\xi_1\xi_2 & \beta\xi_1\xi_3 \\ \nu\xi_2\xi_1 & \lambda + \mu|\xi|^2 + \nu\xi_2^2 & \nu\xi_2\xi_3 \\ \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \lambda + \mu|\xi|^2 + \nu\xi_3^2 \end{vmatrix}$$
$$= (\lambda + \mu|\xi|^2)^2\{(\lambda + \mu|\xi|^2) + \nu|\xi|^2\}.$$

Similar technique, we have $(i\gamma\xi_1)(\lambda + \mu|\xi|^2)^2, -(i\gamma\xi_2)(\lambda + \mu|\xi|^2)^2$ and $(i\gamma\xi_3)(\lambda + \mu|\xi|^2)^2$ for $|\widehat{\mathbb{A}}_{12}|$, $|\widehat{\mathbb{A}}_{13}|$, and $|\widehat{\mathbb{A}}_{14}|$, respectively. Substituting $|\widehat{\mathbb{A}}_{11}|, |\widehat{\mathbb{A}}_{12}|, |\widehat{\mathbb{A}}_{13}|$, and $|\widehat{\mathbb{A}}_{14}|$ to equation (9) we have,

$$\det[\lambda I + \widehat{\mathbb{M}}(\xi)] = (\lambda + \alpha|\xi|^2)^2\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} \quad (10)$$

Furthermore, we determine a matrix adjoint of $[\lambda I + \widehat{\mathbb{M}}(\xi)]$ which is a tranpose matrix of cofactor matrix. Since these matrix is a symmetric matrix, so that the determinant of the matrix hold the properties $|\widehat{\mathbb{A}}_{ij}| = |\widehat{\mathbb{A}}_{ji}|$ for $i, j = 1, 2, 3, 4$. Moreover, we enough only determine the $|\widehat{\mathbb{A}}_{22}|$, $|\widehat{\mathbb{A}}_{23}|$, $|\widehat{\mathbb{A}}_{24}|$, $|\widehat{\mathbb{A}}_{33}|$, $|\widehat{\mathbb{A}}_{34}|$ and $|\widehat{\mathbb{A}}_{44}|$. In fact, for $i = j$ we have

$$|\widehat{\mathbb{A}}_{22}| = (\lambda + \mu|\xi|^2)^2\{\lambda(\lambda + \mu|\xi|^2) + (\xi_2^2 + \xi_3^2)(\lambda\nu + \gamma^2)\},$$
$$|\widehat{\mathbb{A}}_{33}| = (\lambda + \mu|\xi|^2)^2\{\lambda(\lambda + \mu|\xi|^2) + (\xi_1^2 + \xi_3^2)(\lambda\nu + \gamma^2)\},$$
$$|\widehat{\mathbb{A}}_{44}| = (\lambda + \mu|\xi|^2)\ \{\lambda(\lambda + \mu|\xi|^2) + (\xi_1^2 + \xi_2^2)(\lambda\nu + \gamma^2)\}.$$

Employing the same argument, we can find others minors

$$|\widehat{\mathbb{A}}_{23}| = (\lambda + \mu|\xi|^2)\{(\xi_1\xi_2)(\lambda\nu + \gamma^2)\} = |\widehat{\mathbb{A}}_{32}|,$$
$$|\widehat{\mathbb{A}}_{24}| = -(\lambda + \mu|\xi|^2)\{(\xi_1\xi_3)(\lambda\nu + \gamma^2)\} = |\widehat{\mathbb{A}}_{42}|,$$
$$|\widehat{\mathbb{A}}_{34}| = -(\lambda + \mu|\xi|^2)\{(\xi_2\xi_3)(\lambda\nu + \gamma^2)\} = |\widehat{\mathbb{A}}_{32}|.$$

Moreover, we have the cofactors in the following

$$\hat{C}_{11} = (\lambda + \mu|\xi|^2)^2\{(\lambda + \mu|\xi|^2) + \nu|\xi|^2\},$$
$$\hat{C}_{1j} = \hat{C}_{j1} - (i\gamma\xi_{j-1})(\lambda + \mu|\xi|^2)^2,$$
$$\hat{C}_{ij} = \hat{C}_{ji}(\lambda + \mu|\xi|^2)\{\lambda(\lambda + \mu|\xi|^2)\delta_{ij} + (\delta_{ij} - \xi_{i-1}\xi_{j-1})(\lambda\nu + \gamma^2)\}$$

with $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ij} = 1$ for $i = j$ and $i, j = 2, 3, 4$.

4. Proof of Theorem

Throughout this section, we use the notation introduced in the previous section.

4.1 Eigen values

In this subsection, we investigate the eigen values. The solution formula for the model problem has already obtained by Kobayashi [5]. Here, we shall give a slightly detail how to get the solution formula for velocity dan the density. First of all, we determine the determinant of matrix $\det[\mathbf{I}\lambda + \widehat{\mathbb{M}}(\xi)] = 0$. By equation (7), we have

$$\det[\lambda I + \widehat{\mathbb{M}}(\xi)] = 0$$

$$\left(\boldsymbol{\lambda} + \boldsymbol{\mu}|\boldsymbol{\xi}|^2\right)^2\{\boldsymbol{\lambda}^2 + (\boldsymbol{\mu} + \boldsymbol{\nu})|\boldsymbol{\xi}|^2\boldsymbol{\lambda} + \boldsymbol{\gamma}^2|\boldsymbol{\xi}|^2\} = \mathbf{0} \quad \textbf{(11)}$$

From equation (9), we have two possibilities sero values, that are $(\lambda + \mu|\xi|^2)^2 = 0$ or $\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} = 0$. For the first case, we have $\lambda_3 = \lambda_4 = -\mu|\xi|^2$. Furthermore, we will find the eigen values of $\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} = 0$. By using the formula

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

we have

$$\lambda_{1,2} = \frac{-(\mu+\nu)|\xi|^2 \pm |\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}}{2}. \quad (12)$$

In view of equation (10), for $|\xi| \geq \frac{2\gamma}{(\mu+\nu)}$, we have

$$\lambda_1 = \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}$$
$$\lambda_2 = \frac{-(\mu+\nu)}{2}|\xi|^2 - \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}.$$

Meanwhile, for $|\xi| \leq \frac{2\gamma}{(\mu+\nu)}$, we have

$$\lambda_1 = \overline{\lambda_2}\ \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{i}{2}|\xi|\sqrt{4\gamma^2 - (\mu+\nu)^2|\xi|^2}.$$

Moreover, for $|\xi| = \frac{2\gamma}{(\mu+\nu)}$ can be been in Kobayashi [5]. Thus, we may omit the calculation.

4.2 Fourier transform of $\hat{\rho}$ and $\hat{\boldsymbol{v}}$

In this subsection we consider the formula of $\hat{\rho}$ and $\hat{\mathbf{v}}$, density and velocity, respectively. These density and velocity are the result of the model problem (1). First of all, applying div to second equation of (5), we have

$$\frac{\partial}{\partial t}\operatorname{div}\mathbf{v} - \mu\Delta\operatorname{div}\mathbf{v} - \nu\Delta\operatorname{div}\mathbf{v} + \gamma\Delta\rho = 0. \quad (13)$$

Let $D = \operatorname{div} v$, then equation (13) can be written as follows

$$D_t - \omega\Delta D + \gamma\Delta\rho = 0,$$

with $\omega = \mu + \nu$. Recalling first equation of (3),

$$\rho_t = -\gamma\operatorname{div} v,$$

then we can write the equation to be

$$\rho_t = -\gamma D,$$
$$\frac{\rho_t}{\gamma} = -D. \quad (14)$$

Here in after, we differentiate the equation of (14) recpect to t variable and substitute (11) and (12) to the result, we obtain

$$\frac{1}{\gamma}\rho_{tt} = -D_t,$$
$$\frac{1}{\gamma}\rho_{tt} = \gamma\,\Delta\,\rho - \omega\Delta D,$$
$$\frac{1}{\gamma}\rho_{tt} = \gamma\,\Delta\,\rho - \omega\left(-\Delta\frac{\rho_t}{\gamma}\right),$$
$$\rho_{tt} = \gamma^2\,\Delta\,\rho + \omega\Delta\rho_t. \quad (15)$$

Applying Fourier transform to equation (15), we have

$$\hat{\rho}_{tt} + \omega|\xi|^2\hat{\rho}_t + \gamma^2\,|\xi|^2\,\hat{\rho} = 0 \quad (16)$$

with the initial condition

$$\hat{\rho}(\xi,0)=\hat{\rho}_0(\xi),\ \ \hat{\rho}_t(\xi,0)=-i\xi\hat{\mathbf{v}}_0(\xi). \tag{17}$$

Moreover, we have the general solution for equation (16)

$$\hat{\rho}(\xi,t)=c_1e^{\lambda_1(\xi)t}+c_2e^{\lambda_2(\xi)t}, \tag{18}$$

where

$$\lambda_{1,2}=\frac{-\omega|\xi|^2\pm|\xi|\sqrt{\omega^2|\xi|^2-4\gamma^2}}{2}.$$

Substituting the initial condition (17) to (18), we obtain

$$c_1=\frac{\lambda_2(\xi)\hat{\rho}_0(\xi)+i\xi\hat{\mathbf{v}}_{\mathbf{0}}(\xi)}{\lambda_2(\xi)-\lambda_1(\xi)},\qquad c_2=-\frac{\lambda_1(\xi)\hat{\rho}_0(\xi)+i\xi\hat{\mathbf{v}}_0(\xi)}{\lambda_2(\xi)-\lambda_1(\xi)}.$$

Therefore, we have the soltion formula for

$$\hat{\rho}(\xi,t)=\left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)t}-\lambda_1(\xi)e^{\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{\lambda_2(\xi)t}-e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi). \tag{19}$$

Furthermore, we determine te solution formula for $\hat{\mathbf{v}}(\xi,t)$. Employing the same argument in [3, Section 3], firstly applying Fourier transform to the second equation of (5), we obtain

$$\hat{\mathbf{v}}_{\mathrm{t}}-\mu|\xi|^2\hat{\mathbf{v}}-\mathrm{i}\nu\xi_{\mathrm{j}}\sum_{k=1}^{3}\mathrm{i}\xi_{\mathrm{k}}\widehat{v_k}+i\gamma\xi_{\mathrm{j}}\hat{\rho}=0, \tag{20}$$

We can write equation (20) in the following sense,

$$\hat{\mathbf{v}}_t=(-\mu|\xi|^2\mathbf{I}-\nu\xi\xi^T)\hat{\mathbf{v}}-i\gamma\xi\hat{\rho}. \tag{21}$$

Vector $\hat{\mathbf{v}}(\xi,t)$ is a vector which parallel and orthogonal from ξ, so that we can write the vector $\hat{\mathbf{v}}(\xi,t)$ as follows

$$\hat{\mathbf{v}}(\xi,t)=a(\xi,t)\frac{\xi}{|\xi|}+b(\xi,t), \tag{22}$$

Where $b(\xi,t)$ orthogonal to ξ, and $a(\xi,t)$ is a scalar such that $a(\xi,t)=\hat{\mathrm{v}}(\xi,t)\frac{\xi}{|\xi|}$.

Substituting equation (22) to (21), then we have

$$\begin{aligned}\hat{\mathbf{v}}_t(\xi,t)&=(-\mu|\xi|^2\mathrm{I}-\nu\xi\xi^T)\left(a(\xi,t)\frac{\xi}{|\xi|}+b(\xi,t)\right)-i\gamma\xi\hat{\rho},\\ &=-a(\xi,t)\frac{\xi}{|\xi|}(\mu|\xi|^2\mathrm{I}+\nu\xi\xi^T)-b(\xi,t)(\mu|\xi|^2\mathrm{I}+\nu\xi\xi^T)-i\gamma\xi\hat{\rho},\\ &=-a(\xi,t)\left(\mu|\xi|\xi+\nu\frac{\xi\cdot\xi\xi^T}{|\xi|}\right)-b(\xi,t)(\mu|\xi|^2+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}.\end{aligned} \tag{23}$$

Next, we differentiate equation (20) and then substitute the result to the left hand-side of (21), we have

$$a_t(\xi,t)\frac{\xi}{|\xi|}+b_t(\xi,t)=-a(\xi,t)\left(\mu|\xi|\xi+\nu\frac{\xi\cdot\xi\xi^T}{|\xi|}\right)-b(\xi,t)(\mu|\xi|^2+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}.$$

Therefore, we obtain

$$a_t(\xi,t)=\omega|\xi|^2a(\xi,t)-i\gamma|\xi|\hat{\rho},\quad b_t(\xi,t)=-\mu|\xi|^2\,b(\xi,t). \tag{24}$$

By (23) and the initial condition $b(\xi,0)=\left(\mathrm{I}-\frac{\xi\xi^T}{|\xi|^2}\right)\hat{\mathbf{v}}_0(\xi)$, we see that

$$b(\xi,t)=e^{\mu|\xi|^2 t}\left(\mathrm{I}-\frac{\xi\xi^T}{|\xi|^2}\right)\hat{\mathbf{v}}_0(\xi). \quad (25)$$

Also, by applying integrating factor to first equation (24), we obtain

$$a(\xi,t)=e^{-\omega|\xi|^2 t}\left(a(\xi,0)-i\,\gamma|\xi|\int_0^t e^{\omega|\xi|^2 s}\,\hat{\rho}(\xi,s)ds\right)$$

with $a(\xi,0)$ constant.

Furthermore, we will investigate the formula of the second term equation (26). Multiplying (19) by $e^{\omega|\xi|^2 s}$, we have

$$\begin{aligned} e^{\omega|\xi|^2 s}\,\hat{\rho}(\xi,s) &= \left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)s+\omega|\xi|^2 s}-\lambda_1(\xi)e^{\lambda_2(\xi)s+\omega|\xi|^2 s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi) \\ &\quad - i\left(\frac{e^{\lambda_2(\xi)s+\omega|\xi|^2 s}-e^{\lambda_1(\xi)s+\omega|\xi|^2 s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \\ &= \left(\frac{\lambda_2(\xi)e^{-\lambda_2(\xi)s}-\lambda_1(\xi)e^{-\lambda_1(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{-\lambda_1(\xi)s}-e^{-\lambda_2(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \quad (26) \end{aligned}$$

since $\lambda_{1,2}(\xi)+\omega|\xi|^2=-\lambda_{2,1}(\xi)$ and $\lambda_1(\xi)\lambda_2(\xi)=\gamma|\xi|^2$.

By integrating (26) from $0\le s\le t$, we have

$$\begin{aligned} &\int_0^t e^{\omega|\xi|^2 s}\,\hat{\rho}(\xi,s)\,ds \\ &\quad = \left(\frac{-e^{-\lambda_2(\xi)t}-e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi) \\ &\quad + \left(\frac{i}{\gamma|\xi|^2}\right)\left(\frac{\lambda_2(\xi)e^{-\lambda_1(\xi)t}-\lambda_1(\xi)e^{-\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \end{aligned}$$

without loss of generality, we take $a(\xi,0)=0$ so that $e^{\omega|\xi|^2 t}a(\xi,0)=0$. Thus, we have

$$a(\xi,t)=-i\gamma|\xi|\left(\frac{-e^{-\lambda_2(\xi)t}-e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)+\left(\frac{\lambda_2(\xi)e^{-\lambda_2(\xi)t}-\lambda_1(\xi)e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\frac{\xi\hat{\mathbf{v}}_0(\xi)}{|\xi|}. \quad (27)$$

Substituting (24) and (27) to (20), this complete the proof of the **Theorem 1.2**.

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