The TES and ARIMA Methods and Its Simulation

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THE TES AND ARIMA METHODS AND THEIR SIMULATION

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Abstract

We study the triple exponential smoothing (TES) and ARIMA methods to forecast and analyze time series data. Both the methods are suitable for typical data with seasonal increasing and seasonal pattern. The mean absolute error (MAE) is used to obtain the eligible forecasting. To compute the result, the Zaitun and Minitab softwares are then used. The result showed that the TES can be considered to be an alternative method with mean error of the MAE equal to 5.05, and the best model of autoregressive integrated moving average (ARIMA) methods as ARIMA$(1, 1, 1)(0, 0, 1)^{12}$ with MAE of 4.01. Following the pattern of the plot and the results, we conclude that the ARIMA model is more eligible than TES.

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1. Introduction

Many authors have discussed a forecasting data analysis on time series data analysis, such as Henke and Wichern [13], Makridakis et al. [16], Palit and Dobrivoje [2], Wei [18], Montgomery [9], Montgomery et al. [10], Chase and Jacobs [15], Aritonang [14], Aidah [1], Ukhra [3] and Pratikno et al. [7]. Furthermore, the detail of the statistical methods for forecasting on time series data analysis is found in Abraham and Ledolter [5]. Here, the time series tends to exhibit a cyclical pattern that has tendency to repeat itself on a fixed period, and the forecasting is a predicting and estimating of the uncertainty condition in the future time. Following Hanke and Wichern [13], there are some patterns of the time series data, namely (1) seasonal, (2) cycles, (3) trend and (4) irregular. Moreover, Makridakis et al. [16] said that there are two kinds of the method in time series data analysis, namely moving average (autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA)) and smoothing methods (single exponential smoothing, double exponential smoothing, and triple exponential smoothing). Note that ARIMA can be used to model the forecasting data on seasonal and non-seasonal time series data. We then used the small error of the mean absolute error (MAE) to ensure the significant result of the forecasting for p period ahead, and the formula of MAE is given as

$$MAE = \frac{1}{n}\sum_{t=1}^{n} | \hat{Z}_t - Z_t |, \qquad (1)$$

where Z_t is an actual data at period t, $\hat{Z}_t$ is a forecasting data at period t, and n is the number of data. An alternative method of equation (1) is the mean absolute percentage error (MAPE). Note that there are some indicators of the MAPE: (1) $0 < x < 10$ is very good, (2) $10 \le x < 20$ is good, (3) $20 \le x < 50$ is enough, and (4) $x \ge 50$ is bad (Goh and Law [8]). More detail of the exponential smoothing can be found in Makridakis [7] and Makridakis et al. [16]. Here, they described that the single exponential smoothing is suitable for the random and stationary data, the double exponential smoothing (Brown and Holt methods) is eligible for the trend increases of the pattern data, and the triple exponential smoothing is then used on the trend seasonal of the pattern data using three smoothing weights, namely α, β and γ (Makridakis et al. [16]). Moreover, the detail of ARIMA can be also found in Box and Cox [11], Box et al. [12], Montgomery [9] and Montgomery et al. [10].

To produce the forecasting on p period ahead for getting the eligible forecasting of the time series data, we note some steps: (1) plot the actual (original) data to identify the trend of the data (time series data), (2) find the suitable method, (3) give a simulation data using theory and software for getting the forecasting data on p period ahead, and (4) check the significant result using MAE or MAPE.

The introduction is presented in Section 1. The ARIMA is given in Section 2. The simulations are obtained in Section 3. Section 4 then describes the conclusion of the research.

2. The Time Series, ARIMA and TES

Following Makridakis et al. [16], there are four types of pattern of the time series data (Figure 1), namely (1) horizontal (H, see Figure 1(a)), (2) trend (T, see Figure 1(b)), (3) seasonal (S, see Figure 1(c)), and cycles (C, see Figure 1(d)).

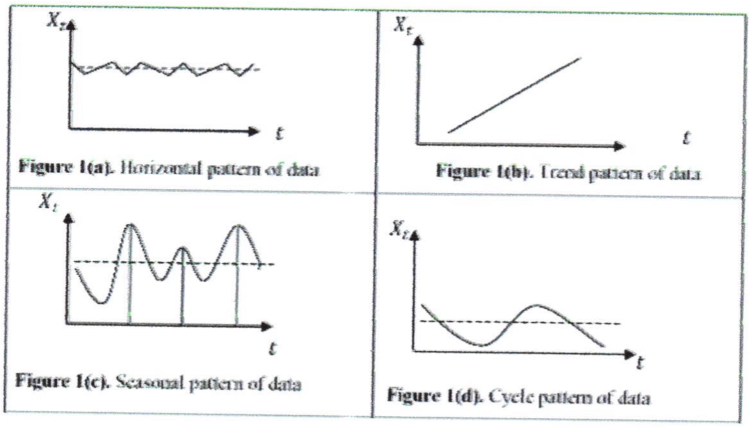


Figure 1(a). Horizontal pattern of data

Figure 1(b). Trend pattern of data

Figure 1(c). Seasonal pattern of data

Figure 1(d). Cycle pattern of data

Figure 1. The types (pattern) of the time series data.

2.1. The ARIMA

Following Box and Cox [11], Box et al. [12] and Makridakis et al. [16], the autoregressive integrated moving average (ARIMA) is used in forecasting time series data. The model is then written as ARIMA (p, d, q), with p is the order of the autoregressive (AR), d is differencing, and q is the order of the moving average (MA). The ARIMA (p, d, q) model of the non-seasonal is then given as $(\phi B)(1 - B)^d X_t = (\theta B)$, where B is backward shift operator. For example, the ARIMA (1, 1, 1) follows 1st differencing (in stationary process) with $p = 1$ and $q = 1$, so the model is written as $(1 - B)(1 - \phi_1 B X_t) = (1 - \theta_1 B)$. This is due to the non-seasonal which can be expressed as $(1 - B)(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)X_t = (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q)X_t$. Furthermore, the seasonal ARIMA model is written as ARIMA $(p, d, q)(P, D, Q)^s$, where (p, d, q) is a part of the non-seasonal and (P, D, Q) is a part of the seasonal model, with s is called the *seasonal period*.

2.1.1. The autoregressive model

Autoregressive (AR) is a linear regression model of the forecasting data. It is a function related to the previous data on time lag. Following Makridakis et al. [16], the autoregressive model (AR) order p, AR(p), is written as

$$X_t = \phi_1 X_{t-1} + \phi_1 X_{t-2} + \cdots + \phi_p X_{t-p} + \varepsilon_t, \tag{2}$$

where ϕ_i are regression coefficients, $i = 1, 2, 3, \ldots, p$, ε_t is error at t, and p is an order of the AR. We then modified it using backward shift operator (B), so equation (2) is expressed as

$$X_t = \phi_1 BX_t + \phi_2 B^2 X_t + \cdots + \phi_p B^p X_t + \varepsilon_t \Rightarrow (\phi B)X_t = \varepsilon_t, \tag{3}$$

with $\phi B = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ is called *operator* of the AR(p). Note that the commonly the order of the AR is $p = 1$ or $p = 2$, namely as AR(1) and AR(2).

2.1.2. The moving average model

Referring to Wei [18], moving average (MA) model with q order, MA(q), is given as

$$X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_p \varepsilon_{t-q};\ \varepsilon_t \sim N(0, \sigma^2), \tag{4}$$

where $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}$ are error terms at $t, t-1, t-2, \ldots, t-q$, ε_t is a white noise (normal distribution), θ_i are regression coefficients, $i: 1, 2, 3, \ldots, q$, and q is the order of the MA. We then re-expressed equation (4) as

$$X_t = (\theta B) \tag{5}$$

with $\theta B = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$ as said operator of the MA(q). Generally, the order of the MA is $q = 1$ or $q = 2$, and it is then written as MA(1) and MA(2).

2.1.3. The ARMA model

We note that a special case of the ARIMA is called autoregressive moving average (ARMA). It is occurred when the value of d is zero (stationer), so it does not need differencing. Generally, the formula of the ARMA model is given as

$$X_t = \phi_1 BX_t + \phi_2 B^2 X_t + \cdots + \phi_p B^p X_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}. \quad (6)$$

Furthermore, we can re-write equation (6) as

$$X_t - \theta_1 BX_t - \phi_2 B^2 X_t - \cdots - \phi_p B^p X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \Rightarrow (\phi B) X_t = (\theta B). \quad (7)$$

2.2. The TES model

In this section, we present the triple exponential smoothing (TES). This is due to the fact that we suspect that there is a little increasing seasonal pattern of the data. Thus, the TES is used to anticipate the suspected seasonal increasing trend of the data. Following Makridakis et al. [16], the three smoothing weighted parameters of the TES, namely α, β and γ, are chosen based on the smallest mean absolute error (MAE) (in many trials). Detailed TES is found in Makridakis et al. [16], and the formula of the TES model is given as

$$\text{Level}: L_t = \alpha(Y_t - S_{t-s}) + (1-\alpha)(L_{t-1} - T_{t-1})$$

$$\text{Trend}: T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)(T_{t-1})$$

$$\text{Seasonal}: S_t = \beta(Y_t - L_t) + (1-\beta)(S_{t-s})$$

$$\text{Forecasting}: \hat{Y}_{t+p} = L_t + pT_t + S_{t-s+p}, \quad (8)$$

where L_t is value of level, α, β and γ are smoothing weights, T_t is an estimation trend, $\alpha = \left(\frac{1}{n}\right)$ is the parameter smoothing $(1 < \alpha < 1)$, β is smoothing constant of trend seasonal, and γ is smoothing constant of trend estimation, S_t is an estimation of seasonal, s is length of seasonal, $\hat{Y}_{t+p}$ is a forecasting data on p period ahead, and p is the period of forecasting.

3. A Simulation Study in the TES and ARIMA

A simulation study is given using the rainfall data from BMKG Cilacap. Here, we simulate 60 data in five years (60 months) data, with some of them as zero (no rain). The original (actual data) plot is presented in Figure 2.

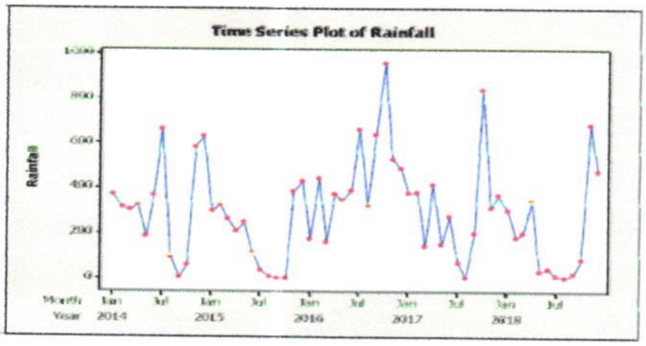


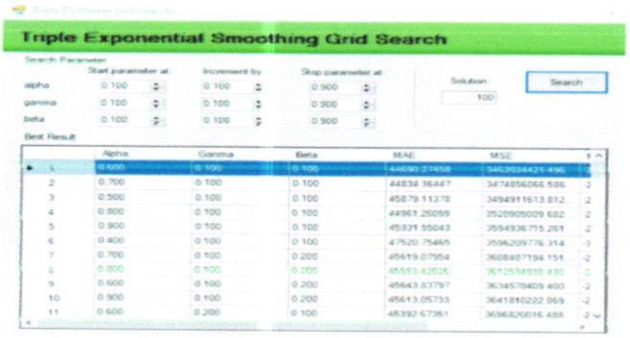
Figure 2. Plot of the rainfall data from BMKG Cilacap.

From Figure 2, we see that there are seasonal patterns on the plot of data and also its trend to increase. Following the types of the pattern in Figure 1, Figure 2 and the previous theory, we, therefore, choose the TES and ARIMA as appropriate models for analyzing this case.

3.1. The simulation study in the TES

In this step, we suspect that there is a little increasing seasonal pattern of the data (not purely seasonal) in big rain season during October-December, which is due to intuitive analysis. Thus, the TES is then used to anticipate this trend in October-December of the data. We first determine the smoothing weighted parameters by choosing the small value of MAE from the output in Table 1.

Table 1. Triple exponential smoothing grid search



Triple Exponential Smoothing Grid Search

Search Parameter

	Start parameter at	Increment by	Stop parameter at
alpha	0.100	0.100	0.900
gamma	0.100	0.100	0.900
beta	0.100	0.100	0.900

Solution: 100

Search

Best Result

	Alpha	Gamma	Beta	MAE	MSE
1	0.600	0.100	0.100	44690.27458	3462024421.496
2	0.700	0.100	0.100	44834.36447	3474856066.586
3	0.500	0.100	0.100	45879.11278	3494911613.812
4	0.800	0.100	0.100	44961.29099	3520905009.682
5	0.900	0.100	0.100	45331.55043	3594936715.261
6	0.400	0.100	0.100	47520.75465	3596209776.314
7	0.700	0.100	0.200	45419.07954	3608487194.151
8	0.800	0.100	0.200	[illegible]	[illegible]
9	0.600	0.100	0.200	45643.83797	3634578409.400
10	0.900	0.100	0.200	45613.05733	3641810222.069
11	0.600	0.200	0.100	45392.67361	3696820016.488

From Table 1, it is clear that the smallest MAE is 44690.27, we then chose the smoothing weighted parameters, $\alpha = 0.6$, $\beta = 0.1$, $\gamma = 0.1$, and period of seasonal $(s) = 12$, with period of predicting is $p = 12$. Note the MAE is used (not the MAPE), this is due to we have a lot of missing data, so the MAPE is not available. Using equation (8) and Zaitun software, we then presented the forecasting of the data in Figure 3.

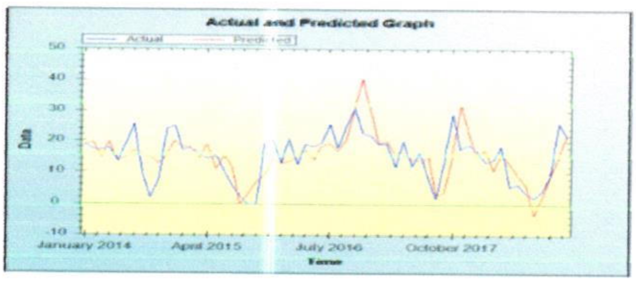


Figure 3. Graph of the forecasting (red line).

Figure 3 showed that the red line (forecasting) is close to the blue line (actual data). It means that their error is small, so we believe that it is significant. Furthermore, the result of the predicting (forecasting) for three months in 2018 and three months (October-December) in 2019 are presented in Table 2.

Table 2. The smoothing level, trend, seasonal and forecasting

No.	Year	Month	Smoothing level (L_t)	Smoothing trend (T_t)	Smoothing seasonal (S_t)	Forecasting $\hat{Y}_t$
Year 2018 (predicting)						
58	**2018**	**Oct**	8.997	-0.384	0.162	9.149
59	**2018**	**Nov**	15.273	0.282	6.847	15.015
60	**2018**	**Dec**	15.275	0.254	6.596	22.168
Year 2019 (really forecasting)						
70	2019	Oct				17.973
71	2019	Nov				24.911
72	2019	Dec				24.914

3.2. The simulation study using ARIMA

This section is very appropriate to the model of the plot data. Intuitively, we see that the pattern is seasonal. Here, we first must check and make stationary the data using partial autocorrelation function (PACF) (see Figure 4).

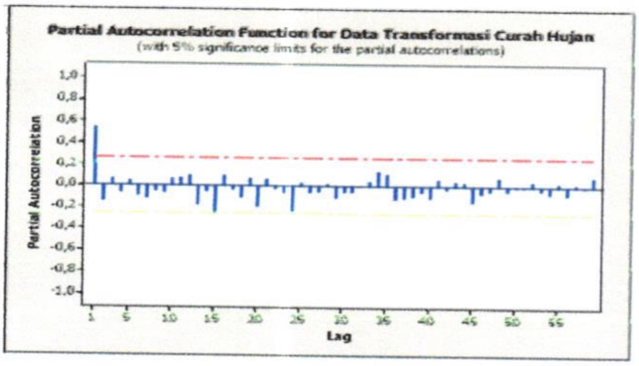


Figure 4. Plot of PACF original data.

Figure 4 showed that the data has not been stationary yet (see *cutoff* at lag 3), so we must do differencing in order to be stationer ($q = 1$). Moreover, we then figured the differencing ($q = 1$) data in Figure 5.

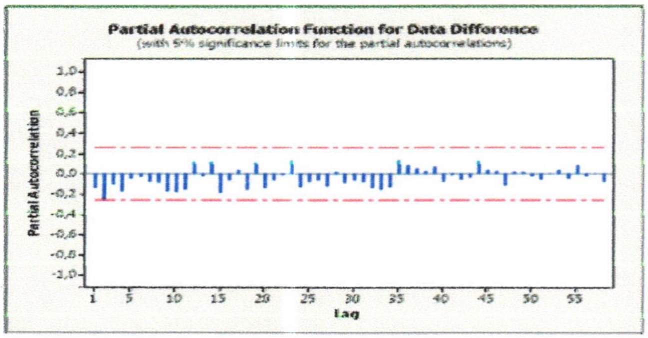


Figure 5. Plot of PACF data after differencing.

We see from Figure 5 that the non-seasonal model is ARIMA(1, 1, 1), (0, 1, 1), (1, 1, 0) and seasonal model $(0, 0, 0)^{12}$, $(1, 0, 1)^{12}$, $(1, 0, 0)^{12}$, $(0, 0, 1)^{12}$, respectively. Using some simulations, we obtained many models of the seasonal ARIMA that are

ARIMA(1, 1, 1) $(0, 0, 0)^{12}$, ARIMA(1, 1, 1) $(1, 0, 1)^{12}$,

ARIMA(1, 1, 1) $(0, 0, 1)^{12}$, ARIMA(1, 1, 1) $(1, 0, 0)^{12}$,

ARIMA(1, 1, 0) $(0, 0, 0)^{12}$, ARIMA(1, 1, 0) $(1, 0, 1)^{12}$,

ARIMA(1, 1, 0)$(1, 0, 0)^{12}$, ARIMA(1, 1, 0) $(0, 0, 1)^{12}$,

ARIMA(0, 1, 1)$(0, 0, 0)^{12}$, ARIMA(0, 1, 1) $(1, 0, 1)^{12}$,

ARIMA(0, 1, 1) $(1, 0, 0)^{12}$, or ARIMA(0, 1, 1)$(0, 0, 1)^{12}$.

Furthermore, using diagnostic check and a lot of tests of the hypothesis testing of the parameter, we got the eligible model of the ARIMA, namely ARIMA(1, 1, 1) $(0, 0, 1)^{12}$. It is chosen from the smallest MAE in Table 3.

Table 3. MAE and seasonal ARIMA

No.	Model	MAE
1	ARIMA(1, 1, 1) $(0, 0, 0)^{12}$	5.196419
2	ARIMA(1, 1, 1) $(0, 0, 1)^{12}$	4.014308
3	ARIMA(0, 1, 1) $(0, 0, 0)^{12}$	5.485637
4	ARIMA(0, 1, 1) $(1, 0, 0)^{12}$	4.969806

Furthermore, the predicting (forecasting) data of the best ARIMA(1, 1, 1) $(0, 0, 1)^{12}$, for 3 months in 2018, are presented in Table 4.

Table 4. The predicting data for 3 months using ARIMA

No.	Year	Month	Actual data	Forecast	Status	Note
10	2018	October	3.02740	3.1264	Close	Accurate
11	2018	November	5.11030	2.9569	Too low	Not accurate
12	2018	December	4.65860	4.6908	Close	Accurate

To compare the accuracy of the predicting data between ARIMA(1, 1, 1) $(0, 0, 1)^{12}$ and the TES, we re-expressed the result of the TES in three months (October-December 2018, see Table 2) as below in Table 5.

Table 5. The predicting data for 3 months using TES

No.	Year	Month	Actual data	Forecast	Status	Note
10	2018	October	3.03740	9.149	Too high	Not accurate
11	2018	November	5.11030	15.015	Too high	Not accurate
12	2018	December	4.65860	22.168	Too high	Not accurate

From both the tables, Table 4 and Table 5, we see that the ARIMA is better than the TES.

4. Conclusion

The research studied the TES and ARIMA methods in forecasting and analyzing time series data. Both the methods are suitable for typical data with seasonal increasing and seasonal pattern. The MAE is used to obtain the eligible forecasting, and the MAPE is not used due to the fact that there are some missing data (zero data). The Zaitun and Minitab softwares are used to compute the result. The result showed that the TES is eligible method with mean error of the MAE as 5.05, and the best model of ARIMA is ARIMA$(1, 1, 1)(0, 0, 1)^{12}$ with MAE as 4.01. Following the pattern of the actual data plot and both the results, we conclude that the ARIMA model is more eligible and significant than the TES.

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