Power of Hypothesis Testing Parameters Shape of the Distributions

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Far East Journal of Mathematical Sciences (FJMS)

http://www.pphmj.com
http://dx.doi.org/10.17654/MS110010015
Volume 110, Number 1, 2019, Pages 15-22 **ISSN: 0972-0871**

POWER OF HYPOTHESIS TESTING PARAMETERS SHAPE OF THE DISTRIBUTIONS

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Abstract

We study the power in testing parameter shape of the distributions, analyze it graphically and provide its application of the power of the tests on the multiple simple regression model (MSRM), namely unrestricted test (UT), restricted test (RT) and pre test-test (PTT). To compute the power and plot of their graphs, *R*-code is used. The results showed that the power of the distribution is influenced by the parameter shapes, and the power of the test of the PTT is a significant choice of the tests among UT and RT.

1. Introduction

Following Wackerly et al. [5], the power and size are defined as probability to reject H_0 under $H_a : \theta = \theta_a$ and under $H_0 : \theta = \theta_0$, in testing hypothesis $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$, with parameter θ, respectively. They are then written as $\pi(\theta_a) = P(\text{reject } H_0 \mid \theta = \theta_a)$ and $\alpha^* = \alpha(\theta_0) = P(\text{reject } H_0 \mid \theta = \theta_0)$. Note that α (level of significant: 0.01, 0.05 and 0.10) is commonly a special case of the $\alpha^* = \alpha(\theta)$.

Received: August 3, 2018; Accepted: October 16, 2018
2010 Mathematics Subject Classification: 62H10, 62E17, 62Q05.
Keywords and phrases: distributions, power of the test, parameter shape.

Many authors have used the power and size to compute the probability integral cumulative distribution function (cdf) of the distributions, such as Pratikno [2], Khan and Pratikno [8] and Khan [9] in testing intercept using non-sample prior information (NSPI). Here, Pratikno [3] and Khan et al. [15] already used the power and size to compute the cdf of the bivariate noncentral *F* (BNCF) distribution of the pre-test test (PTT) in multivariate simple regression model (MSRM), multiple regression model (MRM) and parallel regression model (PRM). Then Khan [9, 10], Khan and Saleh [12-14], Khan and Hoque [11], Saleh [1], Yunus [7], and Yunus and Khan [6] contributed to the research of estimation and hypothesis area in computing the values of the power of the test (PTT). In the context of the hypothesis testing with NSPI, Pratikno [2] already described three tests for testing intercept in the regression models, namely unrestricted test (UT), restricted test (RT) and pre-test test (PTT). In this case, the bivariate noncentral *F* distribution is used to compute the power of the pre-test test (PTT) on the MSRM, MRM and PRM. The formula of the power and size of the tests of the UT, RT and PTT are found in Pratikno [3] in testing one-side hypothesis or two-side hypothesis. Due to the probability integral of the power and size of the PTT is not simple and very complex, so they are computed using R-code. The BNCF is found on Pratikno [2] and Khan et al. [15], and it is clear that the computation of the probability integral of the probability distribution function (pdf) and cdf of the BNCF distribution is very complicated and hard, so they should be numerically computed using R code. A simulation is given by generating randomly data from R-package on some regression models case.

To compute the power of the distribution and its application in the regression models, the steps of the research methodology are (1) to find the sufficient statistics, (2) to determine the rejection area of the distributions using *uniformly most powerful test* (UMPT), (3) to derive the formula of the power of the distributions in testing one-side (or two-side) hypothesis, and (4) to plot the graphs of the power of the three tests UT, RT and PTT conducted using a simulation by generating data from R-package.

The research presented the introduction in Section 1. Analysis of the power of the distributions and their application on the power and size of the tests in MSRM are obtained in Section 2. Section 3 describes the conclusion of the research.

2. The Power of the Distributions

2.1. The power of the Weibull distribution

This subsection presents the formula and graphs of the power in testing parameters shape (δ, β) for one-side hypothesis on the Weibull distribution. To do that, we follow some steps, that are (1) find the sufficiently statistics, (2) determine the rejection area of the Weibull distribution using *uniformly most powerful test* (UMPT), (3) derive the formula of the power and compute the values of power and then plot them. This distribution (Weibull distribution) is often applied in life testing of the components, and it is like exponential and gamma distributions.

Let X be a random variable following the Weibull distribution. Then the cdf of this distribution is given as

$$F(x) = \begin{cases} 1 - e^{-\left(\frac{x}{\delta}\right)^{\beta}}, & x \geq 0, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

with parameter shape $\delta > 0$ and scale parameter $\beta > 0$. Furthermore, the density function is then obtained as

$$f(x) = \frac{dF(x)}{dx} = \begin{cases} \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^{\beta}}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

To compute the power of the distribution, we firstly consider the sufficiently statistics. Here, we use it to find the rejection area. To do this, we first define the *likelihood* function of the Weibull distribution as

$$f(x_1, \ldots, x_n \mid \delta) = g(s, \delta) \cdot h(x_1, \ldots, x_n) \tag{3}$$

with

$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^{\beta}},$$

$$f(x_1, ..., x_n \mid \delta) = \prod f(x_i \mid \delta) = \frac{\beta}{\delta}\left(\frac{1}{\delta}\right)^{\beta-1}\left(\prod_{i=1}^{n} x_i\right)^{\beta-1} e^{-\left(\frac{1}{\delta}\right)^{\beta}\left(\sum_{i=1}^{n} x_i^{\beta}\right)},$$

$$g(s, \delta) = \frac{\beta}{\delta}\left(\frac{1}{\delta}\right)^{\beta-1} e^{-\left(\frac{1}{\delta}\right)^{\beta} s},$$

$$h(x_i) = \prod_{i=1}^{n} (x_i)^{\beta-1}, i = 1, 2, ..., n, \text{ and } s = \sum_{i=1}^{n} x_i^{\beta}.$$

Using mathematical technique, we get $s = \sum_{i=1}^{n} x_i^{\beta}$ to be sufficiently statistics of the parameter δ of the Weibull distribution. To find the rejection region (*RR*), we use UMPT, the *RR* of the Weibull distribution is then given as $P(s > \chi^2_{(2n,\alpha)})$, with s is sufficient statistics and δ is parameter shape of the Weibull distribution. Furthermore, we derive the formula of power of the Weibull distribution for one-side testing hypothesis, $H_0 : \delta = \delta_0$ versus $H_1 : \delta > \delta_1$, given as

$$\pi(\delta) = P(\text{reject } H_0 \mid \text{under } H_1) = P\left(\sum_{i=1}^{n} x_i^{\beta} > k\right) = P\left(\frac{2}{\delta^{\beta}}\sum_{i=1}^{n} x_i^{\beta} > c\right)$$

$$= P\left(\sum_{i=1}^{n} x_i^{\beta} > \chi^2_{(2n,\alpha)}\frac{\delta_0^{\beta}}{2}\right)$$

$$= P\left(\chi^2 > \left(\frac{\delta_0}{\delta}\right)^{\beta}\chi^2_{(2n,\alpha)}\right) \text{ with } c = \chi^2_{(2n,\alpha)}. \tag{4}$$

Following Pratikno [3, 4] (here, $\alpha = 0.1$, $n = 10$, 30 and 40) and using equation (4), we get the graphs of the power for $\alpha = 0.05$ and $n = 20$, $\beta = 2, 3, 4, 5$, on hypothesis testing $H_0 : \delta = \delta_0 = 1$ versus $H_1 : \delta_0 > 1$, presented in Figure 1.

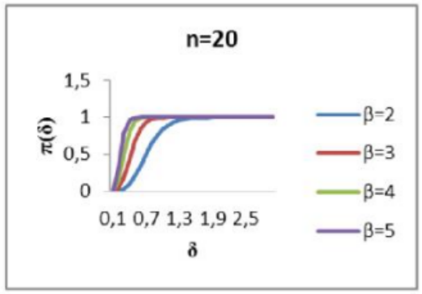


Figure 1. The graphs of power in testing parameter δ at $\alpha = 0.05$.

From Figure 1, it is clear that the graphs of the power tend to increase as the sample size (n) and β increase. Similarly, following Pratikno [3, 4] on $\alpha = 0.01$ and Figure 1 on $\alpha = 0.05$, we see that α has a little significant influence on the curve of the power of the parameter shape, especially when $n = 30$.

2.2. The power of the *t* distribution

Similarly (see Subsection 2.1), we provide graphs for the power in testing parameter shape (ν) for two-side hypothesis, $H_0 : \nu = \nu_0$ versus $H_1 : \nu \neq \nu_0$, on the *t* distribution. Let *T* be a random variable following the *t* distribution. Then the probability density function (pdf) is given as

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\nu}}\left(1+\frac{t^2}{2}\right)^{-\left(\frac{\nu+1}{2}\right)}, \quad -\infty < t < \infty. \tag{5}$$

Furthermore, the formula of power of the *t* distribution for two-side testing hypothesis $H_0 : \nu = \nu_0$ versus $H_1 : \nu \neq \nu_0$, (could be one-side as well), is given as

$$\pi(\nu) = P(\text{reject } H_0 \,|\, \text{under } H_1)$$

$$= 1 - \int_{t_1}^{t_2} f(t)dt = \int_{t_1}^{t_2} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\nu}}\left(1+\frac{t^2}{2}\right)^{-\left(\frac{\nu+1}{2}\right)} dt, \tag{6}$$

where t_1 and t_2 are under H_1. Then the size is given as $\alpha^* = \alpha(\nu) = P(\text{reject } H_0 \,|\, \text{under } H_0)$. Here, we note and write that $\alpha(\nu) = \alpha$. From equation (6) and the definition of the size above, the graphs of the power and size of this distribution are given as

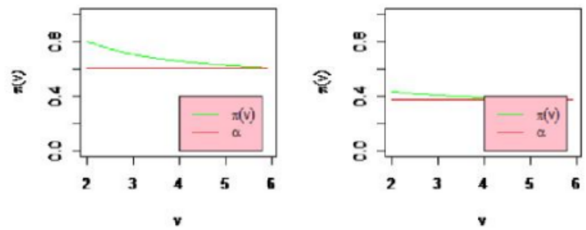


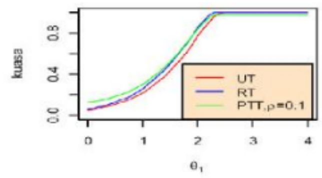
Figure 2. The graphs of power and size for different t_1 and t_2.

We see from Figure 2 that the graphs of the power $\pi(\nu)$ tend to decrease as the parameter ν increases, while the size (α) is constant. Note that the difference between t_1 and t_2 $(\Delta = t_1 - t_2)$, also influenced the curve of the graphs.

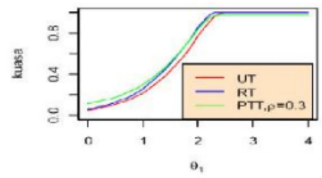
2.3. The power of the UT, RT and PTT on MSRM

Following Pratikno [2], the power of the UT, RT and PTT is used to choose the eligible tests (maximum power and minimum size) in testing intercept on the regression models. Here, the definition and formula of the UT, RT and PTT on MSRM are found in detail in Pratikno [2]. A simulation is given following the previous research of Pratikno [2], and here, we simulate and generate data using R-package as follows: $y_1 = 3 + 2x$, $y_2 = 2 - 3x$, $y_3 = 3 - 2x$ and $y_4 = 6 + 4x$ with $\alpha = 0.05$ for two-side hypothesis in testing parameters regression, we then obtain the graphs of the power of the UT, RT, PTT in Figure 3.

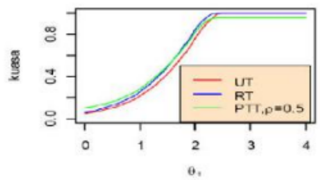
From Figure 3, we see that the PTT tends to lie between UT and RT. It means that the PTT is an alternative choice between them. So, we conclude that the PTT follows the previous research of Pratikno [2].



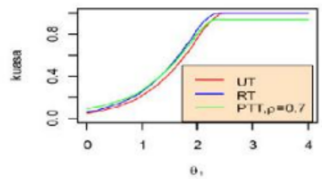
(a) $\theta_2 - 0.5$



(b) $\theta_2 - 0.5$



(c) $\theta_2 = 0.5$



(d) $\theta_2 = 0.5$

Figure 3. The power of the UT, RT, PTT when $\rho = 0.1, 0.3, 0.5, 0.7$ and $\theta_2 = 0.5$.

3. Conclusion

The research studied the power in testing parameter shape of the distributions, analyzed it graphically and provided its application of the power of the tests on the multivariate simple regression model. To compute the power and plot of their graphs, *R*-code is used. The results showed that the power of the distribution is influenced by the parameter shapes, and the power of the test of the PTT is a significant choice of the tests among UT and RT.

Acknowledgement

We acknowledge the excellent financial support from the LPPM UNSOED for carrying out this work.

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