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Preface

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Report from The Organizing Committee

It is indeed my great pleasure and honor to welcome you all to Soedirman's International Conference on Mathematics and Applied Sciences (SICoMAS) 2019. The conference running this year is the first SICoMAS series hosted by Faculty of Mathematics and Natural Sciences Jenderal Soedirman University. As the development of technology and management of world resources for our future based on the innovation in Mathematics and Sciences, this conference takes issue "Innovation in Mathematics and Applied Sciences for better future".

SICoMAS 2019 aims to provide a platform for researchers, lecturers, teachers, students, practitioners, and industrial professionals to share knowledge, exchange ideas, collaborate, and present research results in the fields of Mathematics, Chemistry, Physics, and their applications. Hence, my sincere gratitude goes to our four keynote speakers (Prof. Dr. Hadi Nur from University Teknologi Malaysia, Prof. Dr. Hirokazu Saito from Tokyo University of Science, Dr. Devi Putra , ST, M.Sc. from Pertamina Research and Tecnology, and Uyi Sulaeman, Ph.D. from Jenderal Soedirman University), and our six invited speakers (Prof. Dr. Youtoh Imai from Nishogakusha University, Prof. Riyanto, Ph.D. from Universitas Islam Indonesia, Dr. Moh. Adhib Ulil Absor from Gadjah Mada University, Bambang Hendriya Guswanto, Ph.D, Dadan Hermawan, Ph.D. and Dr. Eng. Mukhtar Effendi, M. Eng. from Jenderal Soedirman University) for sharing their expertise in this conference. My deepest appreciation also goes to our 80 presenters and 7 non presenters for their commitment to participate in this conference.

As the output of this conference, some selected papers in the field of chemistry will be published in Jurnal Molekul which is accredited Sinta 1; and other selected papers in the fields of Mathematics, Physics, Physical Chemistry, and Innovative Chemistry Education will be published in IOP Conference Series Journal. So, I greatly thank Jenderal Soedirman University, all our contributors, and all the members of the committee for the invaluable support that makes this conference a reality.
Finally, I would like to apologize for any short comings found in this conference; and hopefully this two-day conference will be engraved in your memory.

The chair of SICoMAS 2019

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Half-space model problem for a compressible fluid model of Korteweg type with slip boundary condition

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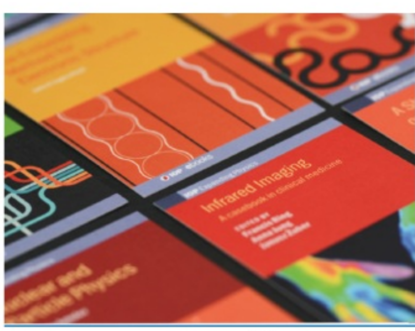

Half-space model problem for a compressible fluid model of Korteweg type with slip boundary condition

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Abstract. In this paper, we consider a half-space model problem for a compressible fluid model of Korteweg type with slip boundary condition and prove the existence of $\mathcal{R}$-bounded solution operator families for the model problem.

1. Introduction

In this paper, we consider a half-space model problem for a compressible fluid model of Korteweg type with slip boundary condition as follows:

$$\begin{cases} \lambda\rho + \mathrm{div}\,\mathbf{u} = d & \text{in } \mathbf{R}^N_+, \\ \lambda\mathbf{u} - \mu\Delta\mathbf{u} - \nu\nabla\mathrm{div}\,\mathbf{u} - \kappa\nabla\Delta\rho = \mathbf{f} & \text{in } \mathbf{R}^N_+, \\ \mathbf{n}\cdot\nabla\rho = g & \text{on } \mathbf{R}^N_0, \\ \partial_N u_j + \partial_j u_N = h_j & \text{on } \mathbf{R}^N_0, \quad j = 1, \dots, N-1, \\ u_N = h_N & \text{on } \mathbf{R}^N_0, \end{cases} \tag{1.1}$$

where $\mathbf{R}^N_+$ and $\mathbf{R}^N_0$, $N \geq 2$, are respectively the upper half-space and its boundary, that is,

$$\begin{aligned} \mathbf{R}^N_+ &= \{x = (x', x_N) \mid x' = (x_1, \dots, x_{N-1}) \in \mathbf{R}^{N-1}, x_N > 0\}, \\ \mathbf{R}^N_0 &= \{x = (x', x_N) \mid x' = (x_1, \dots, x_{N-1}) \in \mathbf{R}^{N-1}, x_N = 0\}, \end{aligned}$$

and also $\mathbf{n} = (0, \dots, 0, -1)^{T}$[5] is the outward unit normal vector on $\mathbf{R}^N_0$.

Here λ is the resolvent parameter varying in $\mathbf{C}_+ = \{z \in \mathbf{C} \mid \Re z > 0\}$, while $\rho = \rho(x)$ and $\mathbf{u} = \mathbf{u}(x) = (u_1(x), \dots, u_N(x))^T$ are respectively the fluid density and the fluid velocity that are unknown functions. The right-hand sides $d = d(x)$, $\mathbf{f} = \mathbf{f}(x) = (f_1(x), \dots, f_N(x))^T$, $g = g(x)$, $h_j = h_j(x)$, and $h_N = h_N(x)$ are given functions. For a scalar-valued function $u = u(x)$ and a

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[5] $\mathbf{M}^T$ denotes the transpose of $\mathbf{M}$.

vector-valued function $\mathbf{v} = \mathbf{v}(x) = (v_1(x), \ldots, v_N(x))^T$, we set for $\partial_k = \partial/\partial x_k$ $(k = 1, \ldots, N)$

$$\begin{aligned} \nabla u &= (\partial_1 u, \ldots, \partial_N u)^T, \quad \Delta u = \textstyle\sum_{k=1}^N \partial_k^2 u, \quad \Delta \mathbf{v} = (\Delta v_1, \ldots, \Delta v_N)^T, \\ \mathrm{div}\, \mathbf{v} &= \textstyle\sum_{k=1}^N \partial_k v_k, \quad \nabla \mathbf{v} = \{\partial_k v_l \mid k, l = 1, \ldots, N\}, \\ \nabla^2 \mathbf{v} &= \{\partial_k \partial_l v_m \mid k, l, m = 1, \ldots, N\}. \end{aligned}$$

In addition, for N-vectors $\mathbf{a} = (a_1, \ldots, a_N)^T$ and $\mathbf{b} = (b_1, \ldots, b_N)^T$,

$$\mathbf{a} \cdot \mathbf{b} = \sum_{k=1}^N a_k b_k.$$

Especially, $\mathbf{n} \cdot \nabla \rho = -\partial_N \rho$. Throughout this paper, the coefficients μ, ν, and κ are positive constants and satisfy the following condition:

$$\left(\frac{\mu + \nu}{2\kappa}\right)^2 - \frac{1}{\kappa} \neq 0 \quad \text{and} \quad \kappa \neq \mu\nu. \tag{1.2}$$

System (1.1) arises from the study of compressible viscous fluids of Korteweg type with slip boundary condition, see e.g. [3]. Korteweg-type models are employed to describe a two-phase mixture model of liquid-gas flow.

To introduce our main result, we introduce the notation.

The set of all natural numbers is denoted by $\mathbf{N}$ and $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$. For $q \in [1, \infty]$, $L_q(\mathbf{R}_+^N)$ and $H_q^m(\mathbf{R}_+^N)$, $m \in \mathbf{N}$, denote respectively the Lebesgue spaces on $\mathbf{R}_+^N$ and the Sobolev spaces on $\mathbf{R}_+^N$. We set $H_q^0(\mathbf{R}_+^N) = L_q(\mathbf{R}_+^N)$ and write the norm of $H_q^n(\mathbf{R}_+^N)$, $n \in \mathbf{N}_0$, by $\|\cdot\|_{H_q^n(\mathbf{R}_+^N)}$. Let X and Y be Banach spaces. Then X^m, $m \in \mathbf{N}$, denotes the m-product space of X, while the norm of X^m is usually denoted by $\|\cdot\|_X$ instead of $\|\cdot\|_{X^m}$ for short. The set of all bounded linear operators from X to Y is denoted by $\mathcal{L}(X, Y)$, and $\mathcal{L}(X)$ is the abbreviation of $\mathcal{L}(X, X)$. For a domain U in $\mathbf{C}$, $Hol(U, \mathcal{L}(X, Y))$ stands for the set of all $\mathcal{L}(X, Y)$-valued holomorphic functions defined on U.

For the right member $(d, \mathbf{f}, g, h_1, \ldots, h_{N-1}, h_N)$, we set

$$\mathcal{X}_q(\mathbf{R}_+^N) = H_q^1(\mathbf{R}_+^N) \times L_q(\mathbf{R}_+^N)^N \times H_q^2(\mathbf{R}_+^N) \times H_q^1(\mathbf{R}_+^N)^{N-1} \times H_q^2(\mathbf{R}_+^N).$$

In addition, for solutions of (1.1), we set

$$\begin{aligned} \mathfrak{A}_q(\mathbf{R}_+^N) &= L_q(\mathbf{R}_+^N)^{N^3+N^2+N+1}, & \mathcal{S}_\lambda \rho &= (\nabla^3 \rho, \lambda^{1/2} \nabla^2 \rho, \lambda \nabla \rho, \lambda^{3/2} \rho); \\ \mathfrak{B}_q(\mathbf{R}_+^N) &= L_q(\mathbf{R}_+^N)^{N^3+N^2+N}, & \mathcal{T}_\lambda \mathbf{u} &= (\nabla^2 \mathbf{u}, \lambda^{1/2} \nabla \mathbf{u}, \lambda \mathbf{u}). \end{aligned}$$

Let $\mathbf{F} = (d, \mathbf{f}, g, h_1, \ldots, h_{N-1}, h_N) \in \mathcal{X}_q(\mathbf{R}_+^N)$ and $\mathcal{R}_\lambda f = (\nabla f, \lambda^{1/2} f)$. Then we define $\mathfrak{X}_q(\mathbf{R}_+^N)$ and $\mathcal{F}_\lambda$ as follows:

$$\begin{aligned} &\mathfrak{X}_q(\mathbf{R}_+^N) = L_q(\mathbf{R}_+^N)^{\mathcal{N}}, \quad \mathcal{F}_\lambda \mathbf{F} = (\mathcal{R}_\lambda d, \mathbf{f}, \mathcal{T}_\lambda g, \mathcal{R}_\lambda h_1, \ldots, \mathcal{R}_\lambda h_{N-1}, \mathcal{T}_\lambda h_N) \in \mathfrak{X}_q(\mathbf{R}_+^N), \\ &\mathcal{N} = (N+1) + N + (N^2 + N + 1) + (N-1)(N+1) + (N^2 + N + 1). \end{aligned}$$

At this point, we introduce the definition of the $\mathcal{R}$-boundedness. Let $\mathrm{sign}(a)$ be the sign function of a. Then the definition is given by

Definition 1.1. *Let X and Y be Banach spaces, and let $r_j(u)$ be the Rademacher functions on* $[0, 1]$. *i.e.*

$$r_j(u) = \mathrm{sign}\, \sin(2^j \pi u) \quad (j \in \mathbf{N}, 0 \leq u \leq 1).$$

A family of operators $\mathcal{T} \subset \mathcal{L}(X,Y)$ is called $\mathcal{R}$-bounded on $\mathcal{L}(X,Y)$, if there exist constants $p \in [1,\infty)$ and $C > 0$ such that the following assertion holds: For each $m \in \mathbf{N}$, $\{T_j\}_{j=1}^m \subset \mathcal{T}$, and $\{f_j\}_{j=1}^m \subset X$, there holds

$$\left(\int_0^1 \left\|\sum_{j=1}^m r_j(u)T_j f_j\right\|_Y^p du\right)^{1/p} \leq C\left(\int_0^1 \left\|\sum_{j=1}^m r_j(u) f_j\right\|_X^p du\right)^{1/p}.$$

The smallest such C is called $\mathcal{R}$-bound of $\mathcal{T}$ on $\mathcal{L}(X,Y)$ and denoted by $\mathcal{R}_{\mathcal{L}(X,Y)}(\mathcal{T})$.

Remark 1.2. (1) The constant C in Definition 1.1 may depend on p.

(2) It is known that $\mathcal{T}$ is $\mathcal{R}$-bounded for any $p \in [1,\infty)$, provided that $\mathcal{T}$ is $\mathcal{R}$-bounded for some $p \in [1,\infty)$. This fact follows from Kahane's inequality, see [4, Theorem 2.4].

Now we state the main result of this paper.

Theorem 1.3. *Let $q \in (1,\infty)$ and assume that μ, ν, and κ are positive constants satisfying 1.2. Then, for any $\lambda \in \mathbf{C}_+$, there exist operators $\mathcal{A}(\lambda)$ and $\mathcal{B}(\lambda)$, with*

$$\begin{aligned} \mathcal{A}(\lambda) &\in Hol(\mathbf{C}_+, \mathcal{L}(\mathfrak{X}_q(\mathbf{R}_+^N), H_q^3(\mathbf{R}_+^N))), \\ \mathcal{B}(\lambda) &\in Hol(\mathbf{C}_+, \mathcal{L}(\mathfrak{X}_q(\mathbf{R}_+^N), H_q^2(\mathbf{R}_+^N)^N)), \end{aligned}$$

such that, for any $\mathbf{F} = (d, \mathbf{f}, g, h_1, \ldots, h_{N-1}, h_N) \in \mathcal{X}_q(\mathbf{R}_+^N)$,

$$(\rho, \mathbf{u}) = (\mathcal{A}(\lambda)\mathcal{F}_\lambda \mathbf{F}, \mathcal{B}(\lambda)\mathcal{F}_\lambda \mathbf{F})$$

is a unique solution to (1.1). In addition, for $n = 0, 1$,

$$\begin{aligned} \mathcal{R}_{\mathcal{L}((\mathfrak{X}_q(\mathbf{R}_+^N), \mathfrak{A}_q(\mathbf{R}_+^N)))}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n (\mathcal{S}_\lambda \mathcal{A}(\lambda)) \,\middle|\, \lambda \in \mathbf{C}_+\right\}\right) &\leq C, \\ \mathcal{R}_{\mathcal{L}((\mathfrak{X}_q(\mathbf{R}_+^N), \mathfrak{B}_q(\mathbf{R}_+^N)))}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n (\mathcal{T}_\lambda \mathcal{B}(\lambda)) \,\middle|\, \lambda \in \mathbf{C}_+\right\}\right) &\leq C, \end{aligned}$$

where $C = C(N, q, \mu, \nu, \kappa)$ is a positive constant.

This paper is organized as follows: The next section introduces a reduced system for (1.1) and shows that Theorem 1.3 follows from the main result for the reduced system. In Section 3, we calculate representation formulas for solutions of the reduced system by using the partial Fourier transform with respect to $x' = (x_1, \ldots, x_{N-1})$ and its inverse transform. Section 4 proves our main theorem for the reduced system by results obtained in Section 3.

2. Reduced system

Set $u_j = v_j$ $(j = 1, \ldots, N-1)$ and $u_N = v_N + h_N$ in (1.1). Then $\mathbf{v} = (v_1, \ldots, v_N)^T$ satisfies

$$\begin{cases} \lambda\rho + \mathrm{div}\ \mathbf{v} = \tilde{d} & \text{in } \mathbf{R}_+^N, \\ \lambda\mathbf{v} - \mu\Delta\mathbf{v} - \nu\nabla\mathrm{div}\ \mathbf{v} - \kappa\nabla\Delta\rho = \tilde{\mathbf{f}} & \text{in } \mathbf{R}_+^N, \\ \mathbf{n}\cdot\nabla\rho = g & \text{on } \mathbf{R}_0^N, \\ \partial_N v_j + \partial_j v_N = \tilde{h}_j & \text{on } \mathbf{R}_0^N, \quad j = 1, \ldots, N-1 \\ v_N = 0 & \text{on } \mathbf{R}_0^N, \end{cases} \tag{2.1}$$

where

$$\begin{aligned} \tilde{d} &= d - \partial_N h_N, \\ \tilde{\mathbf{f}} &= \mathbf{f} - (-\nu\partial_1\partial_N h_N, \ldots, -\nu\partial_{N-1}\partial_N h_N, \lambda h_N - \mu\Delta h_N - \nu\partial_N^2 h_N)^T, \\ \tilde{h}_j &= h_j - \partial_j h_N. \end{aligned}$$

Furthermore, similarly to the last part of [2, Section 2], we can reduce (2.1) to the following system:

$$\begin{cases} \lambda\rho + \mathrm{div}\,\mathbf{u} = 0 & \text{in } \mathbf{R}^N_+, \\ \lambda\mathbf{u} - \mu\Delta\mathbf{u} - \nu\nabla\mathrm{div}\,\mathbf{u} - \kappa\nabla\Delta\rho = 0 & \text{in } \mathbf{R}^N_+, \\ \mathbf{n}\cdot\nabla\rho = g & \text{on } \mathbf{R}^N_0, \\ \partial_N u_j + \partial_j u_N = h_j & \text{on } \mathbf{R}^N_0, \quad j = 1, \ldots, N-1, \\ u_N = 0 & \text{on } \mathbf{R}^N_0 \end{cases} \tag{2.2}$$

Let us define function spaces for (2.2). Set $\mathcal{Y}_q(\mathbf{R}^N_+) = H^2_q(\mathbf{R}^N_+) \times H^1_q(\mathbf{R}^N_+)^{N-1}$ and let $\mathbf{G} = (g, h_1, \ldots, h_{N-1}) \in \mathcal{Y}_q(\mathbf{R}^N_+)$. Then $\mathfrak{Y}_q(\mathbf{R}^N_+)$ and $\mathcal{G}_\lambda$ are defined as follows:

$$\mathfrak{Y}_q(\mathbf{R}^N_+) = L_q(\mathbf{R}^N_+)^M, \quad \mathcal{G}_\lambda\mathbf{G} = (\mathcal{T}_\lambda g, \mathcal{R}_\lambda h_1, \ldots, \mathcal{R}_\lambda h_{N-1}) \in \mathfrak{Y}_q(\mathbf{R}^N_+),$$
$$M = (N^2 + N + 1) + (N-1)(N+1).$$

As is discussed in [2, Section 2], it suffices to prove the following theorem in order to complete the proof of Theorem 1.3.

Theorem 2.1. *Let $q \in (1, \infty)$ and assume that μ, ν, and κ are positive constants satisfying 1.2. Then, for any $\lambda \in \mathbf{C}_+$, there exist operators $\widetilde{\mathcal{A}}(\lambda)$ and $\widetilde{\mathcal{B}}(\lambda)$, with*

$$\begin{aligned} \widetilde{\mathcal{A}}(\lambda) &\in Hol(\mathbf{C}_+, \mathcal{L}(\mathfrak{Y}_q(\mathbf{R}^N_+), H^3_q(\mathbf{R}^N_+))), \\ \widetilde{\mathcal{B}}(\lambda) &\in Hol(\mathbf{C}_+, \mathcal{L}(\mathfrak{Y}_q(\mathbf{R}^N_+), H^2_q(\mathbf{R}^N_+)^N)), \end{aligned}$$

such that, for any $\mathbf{G} = (g, h_1, \ldots, h_{N-1}) \in \mathcal{Y}_q(\mathbf{R}^N_+)$,

$$(\rho, \mathbf{u}) = (\widetilde{\mathcal{A}}(\lambda)\mathcal{G}_\lambda\mathbf{G}, \widetilde{\mathcal{B}}(\lambda)\mathcal{G}_\lambda\mathbf{G})$$

is a unique solution to 2.2. In addition, for $n = 0, 1$,

$$\begin{aligned} \mathcal{R}_{\mathcal{L}((\mathfrak{Y}_q(\mathbf{R}^N_+), \mathfrak{A}_q(\mathbf{R}^N_+)))}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n \left(\mathcal{S}_\lambda\widetilde{\mathcal{A}}(\lambda)\right) \middle| \lambda \in \mathbf{C}_+\right\}\right) &\le C, \\ \mathcal{R}_{\mathcal{L}((\mathfrak{Y}_q(\mathbf{R}^N_+), \mathfrak{B}_q(\mathbf{R}^N_+)))}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n \left(\mathcal{T}_\lambda\widetilde{\mathcal{B}}(\lambda)\right) \middle| \lambda \in \mathbf{C}_+\right\}\right) &\le C, \end{aligned}$$

where $C = C(N, q, \mu, \nu, \kappa)$ is a positive constant.

Remark 2.2. The following sections are devoted to the proof of the existence of $\mathcal{R}$-bounded solution operator families stated in Theorem 2.1. The uniqueness of solutions follows from the existence of solutions for the dual problem.

3. Representation formulas for solutions

In this section, following [2, Subsection 3.1], we compute representation formulas for solutions of (2.2). To this end, let us define the partial Fourier transform $\widehat{u}$ of $u = u(x', x_N)$ with respect to $x' = (x_1, \ldots, x_N)$ and its inverse transform as follows:

$$\begin{aligned} \widehat{u} = \widehat{u}(x_N) = \widehat{u}(\xi', x_N) &= \int_{\mathbf{R}^{N-1}} e^{-ix'\cdot\xi'} u(x', x_N)\, dx', \\ \mathcal{F}^{-1}_{\xi'}\left[\widehat{u}(\xi', x_N)\right](x') &= \frac{1}{(2\pi)^{N-1}} \int_{\mathbf{R}^{N-1}} e^{ix'\cdot\xi'} \widehat{u}(\xi', x_N)\, d\xi', \end{aligned}$$

where $\xi' = (\xi_1, \ldots, \xi_{N-1}) \in \mathbf{R}^{N-1}$.

Let $\varphi = \text{div } \mathbf{u}$. Applying the partial Fourier transform to 2.2 yields the following ordinary differential equations:

$$\begin{aligned} \lambda\widehat{\rho} + \widehat{\varphi} &= 0, \quad x_N > 0, && (3.1)\\ \lambda\widehat{u}_j - \mu(\partial_N^2 - |\xi'|^2)\widehat{u}_j - \nu i\xi_j\widehat{\varphi} - \kappa i\xi_j(\partial_N^2 - |\xi'|^2)\widehat{\rho} &= 0, \quad x_N > 0, && (3.2)\\ \lambda\widehat{u}_N - \mu(\partial_N^2 - |\xi'|^2)\widehat{u}_N - \nu\partial_N\widehat{\varphi} - \kappa\partial_N(\partial_N^2 - |\xi'|^2)\widehat{\rho} &= 0, \quad x_N > 0, && (3.3)\end{aligned}$$

with the boundary conditions:

$$\begin{aligned} \partial_N\widehat{\rho}(0) &= -\widehat{g}(0), && (3.4)\\ \partial_N\widehat{u}_j(0) + i\xi_j\widehat{u}_N(0) &= \widehat{h}_j(0), \quad j = 1, \dots, N-1, && (3.5)\\ \widehat{u}_N(0) &= 0. && (3.6)\end{aligned}$$

We then see from (3.1)-(3.3) that

$$\begin{aligned} P_\lambda(\partial_N)\widehat{\varphi} &= 0, && (3.7)\\ (\partial_N^2 - \omega_\lambda^2)P_\lambda(\partial_N)\widehat{u}_J &= 0, \quad J = 1, \dots, N, && (3.8)\end{aligned}$$

where we have set

$$P_\lambda(t) = \lambda^2 - \lambda(\mu+\nu)(t^2 - |\xi'|^2) + \kappa(t^2 - |\xi'|^2)^2, \quad \omega_\lambda = \sqrt{|\xi'|^2 + \frac{\lambda}{\mu}}.$$

Here we have chosen a brunch cut along the negative real axis and a branch of the square root so that $\Re\sqrt{z} > 0$ for $z \in \mathbf{C} \setminus (-\infty, 0]$.

Remark 3.1. Under Condition (1.2), the four roots of $P_\lambda(t)$ are given by $\pm t_1$ and $\pm t_2$, where

$$t_k = \sqrt{|\xi'|^2 + s_k\lambda} \quad (k = 1, 2)$$

for complex numbers s_k $(k = 1, 2)$, depending only on μ, ν, and κ, which satisfy $\Re s_k > 0$ $(k = 1, 2)$ and

$$s_1 \neq s_2, \quad s_1 \neq \mu^{-1}, \quad s_2 \neq \mu^{-1}.$$

For more detail, we refer to [1, Section 3].

In what follows, $j = 1, \dots, N-1$ and $J = 1, \dots, N$. In view of (3.7), (3.8), and Remark 3.1, we look for solutions $\widehat{u}_J$ $(J = 1, \dots, N)$ and $\widehat{\varphi}$ of the forms:

$$\begin{aligned} \widehat{u}_J &= \alpha_J e^{-\omega_\lambda x_N} + \beta_J(e^{-t_1 x_N} - e^{-\omega_\lambda x_N}) + \gamma_J(e^{-t_2 x_N} - e^{-\omega_\lambda x_N}), && (3.9)\\ \widehat{\varphi} &= \sigma e^{-t_1 x_N} + \tau e^{-t_2 x_N}. && (3.10)\end{aligned}$$

One then obtains from $\varphi = \text{div } \mathbf{u}$ and 3.1-3.3

$$i\xi' \cdot \alpha' - i\xi' \cdot \beta' - i\xi' \cdot \gamma' - \omega_\lambda\alpha_N + \omega_\lambda\beta_N + \omega_\lambda\gamma_N = 0, \quad (3.11)$$

$$\beta_j = -\frac{i\xi_j}{t_1}\beta_N, \quad \gamma_j = -\frac{i\xi_j}{t_2}\gamma_N, \quad (3.12)$$

$$\sigma = -\left(\frac{t_1^2 - |\xi'|^2}{t_1}\right)\beta_N, \quad \tau = -\left(\frac{t_2^2 - |\xi'|^2}{t_2}\right)\gamma_N, \quad (3.13)$$

where $i\xi'\cdot a' = \sum_{j=1}^{N-1} i\xi_j a_j$ for $a \in \{\alpha,\beta,\gamma\}$. In addition, we insert (3.9) and (3.10) into (3.4)-(3.6) together with (3.1) in order to obtain

$$\begin{aligned} (t_1^2-|\xi'|^2)\beta_N + (t_2^2-|\xi'|^2)\gamma_N &= \lambda\widehat{g}(0), && (3.14)\\ -\omega_\lambda\alpha_j + (-t_1+\omega_\lambda)\beta_j + (-t_2+\omega_\lambda)\gamma_j &= \widehat{h}_j(0), && (3.15)\\ \alpha_N &= 0. && (3.16) \end{aligned}$$

Let us insert (3.12) into (3.15) to see

$$\alpha_j = -\frac{1}{\omega_\lambda}\left\{\widehat{h}_j(0) + \frac{i\xi_j}{t_1}(-t_1+\omega_\lambda)\beta_N + \frac{i\xi_j}{t_2}(-t_2+\omega_\lambda)\gamma_N\right\}. \tag{3.17}$$

This relation furnishes

$$i\xi'\cdot\alpha' = -\frac{1}{\omega_\lambda}\left\{i\xi'\cdot\widehat{h}'(0) - \frac{|\xi'|^2}{t_1}(-t_1+\omega_\lambda)\beta_N - \frac{|\xi'|^2}{t_2}(-t_2+\omega_\lambda)\gamma_N\right\}, \tag{3.18}$$

where $i\xi'\cdot\widehat{h}'(0) = \sum_{k=1}^{N-1} i\xi_j\widehat{h}_k(0)$. On the other hand, (3.12) yields

$$i\xi'\cdot\beta' = \frac{|\xi'|^2}{t_1}\beta_N, \quad i\xi'\cdot\gamma' = \frac{|\xi'|^2}{t_2}\gamma_N.$$

Inserting these relations, (3.16), and (3.18) into (3.11), we have

$$\beta_N + \gamma_N = \mu\lambda^{-1} i\xi'\cdot\widehat{h}'(0),$$

where we have used the relation $\omega_\lambda^2 - |\xi'|^2 = \mu^{-1}\lambda$. In addition, it follows from (3.14) and $t_k^2 - |\xi'|^2 = s_k\lambda$ $(k=1,2)$ that

$$s_1\beta_N + s_2\gamma_N = \widehat{g}(0).$$

One solves the last two equations in order to obtain

$$\begin{aligned} \beta_N &= \tfrac{1}{s_2-s_1}\left(-\widehat{g}(0) + s_2\mu\lambda^{-1} i\xi'\cdot\widehat{h}'(0)\right),\\ \gamma_N &= \tfrac{1}{s_2-s_1}\left(\widehat{g}(0) - s_1\mu\lambda^{-1} i\xi'\cdot\widehat{h}'(0)\right). \end{aligned} \tag{3.19}$$

Thus we have together with (3.16)

$$\widehat{u}_N = \beta_N(e^{-t_1x_N} - e^{-\omega_\lambda x_N}) + \gamma_N(e^{-t_2x_N} - e^{-\omega_\lambda x_N}).$$

In addition, we have by (3.12) and (3.17)

$$\begin{aligned} \widehat{u}_j &= -\tfrac{1}{\omega_\lambda}\left\{\widehat{h}_j(0) + \tfrac{i\xi_j}{t_1}(-t_1+\omega_\lambda)\beta_N + \tfrac{i\xi_j}{t_2}(-t_2+\omega_\lambda)\gamma_N\right\}e^{-\omega_\lambda x_N}\\ &\quad - \tfrac{i\xi_j}{t_1}\beta_N(e^{-t_1x_N} - e^{-\omega_\lambda x_N}) - \tfrac{i\xi_j}{t_2}\gamma_N(e^{-t_2x_N} - e^{-\omega_\lambda x_N}), \end{aligned}$$

while we have by 3.1, 3.13, and $t_k^2 - |\xi'|^2 = \lambda s_k$ $(k=1,2)$

$$\widehat{\rho} = -\lambda^{-1}\widehat{\varphi} = \frac{s_1}{t_1}\beta_N e^{-t_1x_N} + \frac{s_2}{t_2}\gamma_N e^{-t_2x_N}.$$

Finally, setting $\rho = \mathcal{F}_{\xi'}^{-1}[\widehat{\rho}(\xi',x_N)](x')$ and $u_J = \mathcal{F}_{\xi'}^{-1}[\widehat{u}_J(\xi',x_N)](x')$, we see that $(\rho,\mathbf{u})$ solves (2.2).

4. Proof of Theorem 2.1

Throughout this section, we use the notation introduced in the previous section.

4.1. Solution operators

Let us define

$$\mathcal{M}_0(x_N) = \frac{e^{-t_2 x_N} - e^{-t_1 x_N}}{t_2 - t_1}, \quad \mathcal{M}_l(x_N) = \frac{e^{-t_l x_N} - e^{-\omega_\lambda x_N}}{t_2 - t_1} \quad (l = 1, 2).$$

Noting $s_2 - s_1 = \lambda^{-1}(t_2^2 - t_1^2)$, we can write (3.19) as

$$\begin{aligned} \beta_N &= -\left(\tfrac{\lambda}{t_2^2 - t_1^2}\right)\widehat{g}(0) + \textstyle\sum_{k=1}^{N-1}\left(\tfrac{s_2 \mu i \xi_k}{t_2^2 - t_1^2}\right)\widehat{h}_k(0), \\ \gamma_N &= \left(\tfrac{\lambda}{t_2^2 - t_1^2}\right)\widehat{g}(0) - \textstyle\sum_{k=1}^{N-1}\left(\tfrac{s_1 \mu i \xi_k}{t_2^2 - t_1^2}\right)\widehat{h}_k(0). \end{aligned} \tag{4.1}$$

One constructs from now on solution operators associated with ρ, u_j $(j = 1, \dots, N-1)$, and u_N that are solutions of (2.2) obtained in the previous section. Let us start with ρ. Note that

$$\rho = \mathcal{F}_{\xi'}^{-1}\left[\frac{s_2}{t_2}\gamma_N(e^{-t_2 x_N} - e^{-t_1 x_N})\right](x') + \mathcal{F}_{\xi'}^{-1}\left[\left(\frac{s_1}{t_1}\beta_N + \frac{s_2}{t_2}\gamma_N\right)e^{-t_1 x_N}\right](x').$$

We have by (4.1)

$$\begin{aligned} &\tfrac{s_2}{t_2}\gamma_N(e^{-t_2 x_N} - e^{-t_1 x_N}) \\ &= \tfrac{s_2 \lambda}{t_2(t_2 + t_1)}\widehat{g}(0)\mathcal{M}_0(x_N) - \textstyle\sum_{k=1}^{N-1}\tfrac{s_1 s_2 \mu i \xi_k}{t_2(t_2 + t_1)}\widehat{h}_k(0)\mathcal{M}_0(x_N), \end{aligned}$$

while we have by (3.19)

$$\frac{s_1}{t_1}\beta_N + \frac{s_2}{t_2}\gamma_N = \tfrac{1}{s_2 - s_1}\left(\textstyle\sum_{l=1}^{2}(-1)^l \tfrac{s_l}{t_l}\right)\widehat{g}(0) + \tfrac{s_1 s_2}{s_2 - s_1}\left(\tfrac{1}{t_1} - \tfrac{1}{t_2}\right)\mu\lambda^{-1} i\xi' \cdot \widehat{h}'(0).$$

Combining the last relation with

$$\frac{1}{t_1} - \frac{1}{t_2} = \frac{t_2 - t_1}{t_1 t_2} = \frac{\lambda(s_2 - s_1)}{t_1 t_2 (t_2 + t_1)}$$

furnishes

$$\frac{s_1}{t_1}\beta_N + \frac{s_2}{t_2}\gamma_N = \frac{1}{s_2 - s_1}\left(\sum_{l=1}^{2}(-1)^l \frac{s_l}{t_l}\right)\widehat{g}(0) + \frac{s_1 s_2 \mu}{t_1 t_2 (t_2 + t_1)} i\xi' \cdot \widehat{h}'(0).$$

Summing up the above computations, we see that

$$\begin{aligned} \rho = \; & s_2 \mathcal{F}_{\xi'}^{-1}\left[\tfrac{\lambda}{t_2(t_2 + t_1)}\mathcal{M}_0(x_N)\widehat{g}(\xi', 0)\right](x') - s_1 s_2 \mu \textstyle\sum_{k=1}^{N-1}\mathcal{F}_{\xi'}^{-1} \\ & \left[\tfrac{i\xi_k}{t_2(t_2 + t_1)}\mathcal{M}_0(x_N)\widehat{h}_k(\xi', 0)\right](x') + \tfrac{1}{s_2 - s_1}\textstyle\sum_{l=1}^{2}(-1)^l s_l \mathcal{F}_{\xi'}^{-1}\left[\tfrac{1}{t_l}e^{-t_1 x_N}\widehat{g}(\xi', 0)\right](x') \\ & + s_1 s_2 \mu \textstyle\sum_{k=1}^{N-1}\mathcal{F}_{\xi'}^{-1}\left[\tfrac{i\xi_k}{t_1 t_2 (t_2 + t_1)}e^{-t_1 x_N}\widehat{h}_k(\xi', 0)\right](x') \\ =: \; & \mathcal{A}(\lambda)(g, h_1, \dots, h_{N-1}). \end{aligned}$$

Next we consider u_j $(j = 1, \dots, N-1)$ and u_N. They are respectively given by

$$\begin{aligned} u_j &= -\mathcal{F}^{-1}_{\xi'}\left[\frac{1}{\omega_\lambda}e^{-\omega_\lambda x_N}\widehat{h}_j(\xi',0)\right](x') \\ &+\frac{1}{s_2-s_1}\sum_{l=1}^2(-1)^l\mathcal{F}^{-1}_{\xi'}\left[\frac{i\xi_j(t_l-\omega_\lambda)}{t_l\omega_\lambda}e^{-\omega_\lambda x_N}\widehat{g}(\xi',0)\right](x') \\ &+\frac{s_1s_2\mu}{s_2-s_1}\sum_{l=1}^2\sum_{k=1}^{N-1}(-1)^l\left(\frac{s_l-\mu^{-1}}{s_l}\right)\mathcal{F}^{-1}_{\xi'}\left[\frac{\xi_j\xi_k}{t_l\omega_\lambda(t_l+\omega_\lambda)}e^{-\omega_\lambda x_N}\widehat{h}_k(\xi',0)\right](x') \\ &-\sum_{l=1}^2(-1)^l\mathcal{F}^{-1}_{\xi'}\left[\frac{i\xi_j\lambda}{t_l(t_2+t_1)}\mathcal{M}_l(x_N)\widehat{g}(\xi',0)\right](x') \\ &-s_1s_2\mu\sum_{l=1}^2\sum_{k=1}^{N-1}(-1)^l\frac{1}{s_l}\mathcal{F}^{-1}_{\xi'}\left[\frac{\xi_j\xi_k}{t_l(t_2+t_1)}\mathcal{M}_l(x_N)\widehat{h}_k(\xi',0)\right](x') \\ &=: \mathcal{B}_j(\lambda)(g,h_1,\dots,h_{N-1}) \end{aligned}$$

and

$$\begin{aligned} u_N &= \sum_{l=1}^2(-1)^l\mathcal{F}^{-1}_{\xi'}\left[\frac{\lambda}{t_2+t_1}\mathcal{M}_l(x_N)\widehat{g}(\xi',0)\right](x') \\ &-s_1s_2\mu\sum_{k=1}^{N-1}\sum_{l=1}^2(-1)^l\frac{1}{s_l}\mathcal{F}^{-1}_{\xi'}\left[\frac{i\xi_k}{t_2+t_1}\mathcal{M}_l(x_N)\widehat{h}_k(\xi',0)\right](x') \\ &=: \mathcal{B}_N(\lambda)(g,h_1,\dots,h_{N-1}). \end{aligned}$$

This completes the construction of solution operators.

4.2. Classes of symbols

To construct $\mathcal{R}$-bounded solution operator families from $\mathcal{A}(\lambda)$, $\mathcal{B}_j(\lambda)$ $(j = 1, \dots, N-1)$, and $\mathcal{B}_N(\lambda)$ as above, we introduce two classes of symbols. Let $m(\xi', \lambda)$ be a function, defined on $(\mathbf{R}^{N-1}\setminus\{0\})\times\mathbf{C}_+$, that is infinitely many times differentiable with respect to $\xi' = (\xi_1, \dots, \xi_{N-1})$ and holomorphic with respect to λ. For any multi-index $\alpha' = (\alpha_1, \dots, \alpha_{N-1}) \in \mathbf{N}_0^{N-1}$, let us define

$$\partial^{\alpha'}_{\xi'} = \frac{\partial^{|\alpha'|}}{\partial\xi_1^{\alpha_1}\dots\partial\xi_{N-1}^{\alpha_{N-1}}}, \quad |\alpha'| = \alpha_1 + \dots + \alpha_{N-1}.$$

If there exists a real number r such that for any multi-index $\alpha' = (\alpha_1, \dots, \alpha_{N-1}) \in \mathbf{N}_0^{N-1}$ and $(\xi', \lambda) \in (\mathbf{R}^{N-1}\setminus\{0\})\times\mathbf{C}_+$

$$\left|\partial^{\alpha'}_{\xi'}\left(\left(\lambda\frac{d}{d\lambda}\right)^n m(\xi',\lambda)\right)\right| \leq C(|\lambda|^{1/2}+|\xi'|)^{r-|\alpha'|} \quad (n = 0, 1)$$

with some positive constant C depending on at most N, r, α', μ, ν, and κ, then $m(\xi', \lambda)$ is called a multiplier of order r with type 1. If there exists a real number r such that for any multi-index $\alpha' = (\alpha_1, \dots, \alpha_{N-1}) \in \mathbf{N}_0^{N-1}$ and $(\xi', \lambda) \in (\mathbf{R}^{N-1}\setminus\{0\})\times\mathbf{C}_+$

$$\left|\partial^{\alpha'}_{\xi'}\left(\left(\lambda\frac{d}{d\lambda}\right)^n m(\xi',\lambda)\right)\right| \leq C(|\lambda|^{1/2}+|\xi'|)^{r}|\xi'|^{-|\alpha'|} \quad (n = 0, 1)$$

with some positive constant C depending on at most at most N, r, α', μ, ν, and κ, then $m(\xi', \lambda)$ is called a multiplier of order r with type 2.

Here and subsequently, we denote the set of all symbols of order r with type j on $(\mathbf{R}^{N-1}\setminus\{0\})\times\mathbf{C}_+$ by $\mathbb{M}_{r,j}(\mathbf{C}_+)$. For instance,

$$\xi_k/|\xi'| \in \mathbb{M}_{0,2}(\mathbf{C}_+), \quad \xi_k, \lambda^{1/2} \in \mathbb{M}_{1,1}(\mathbf{C}_+) \quad (k = 1, \dots, N-1),$$

and also $|\xi'|^2, \lambda \in \mathbb{M}_{2,1}(\mathbf{C}_+)$. One notes that $\mathbb{M}_{r,j}(\mathbf{C}_+)$ are vector spaces on $\mathbf{C}$ and that the following fundamental properties hold (cf. [5, Lemma 5.1]).

Lemma 4.1. *Let* $r_1, r_2 \in \mathbf{R}$.

(1) Given $l_j \in \mathbb{M}_{r_j,1}(\mathbf{C}_+)$ $(j = 1, 2)$*, we have* $l_1 l_2 \in \mathbb{M}_{r_1+r_2,1}(\mathbf{C}_+)$.

(2) Given $m_j \in \mathbb{M}_{r_j,j}(\mathbf{C}_+)$ $(j = 1, 2)$*, we have* $m_1 m_2 \in \mathbb{M}_{r_1+r_2,2}(\mathbf{C}_+)$.

(3) Given $n_j \in \mathbb{M}_{r_j,2}(\mathbf{C}_+)$ $(j = 1, 2)$*, we have* $n_1 n_2 \mathbb{M}_{r_1+r_2,2}(\mathbf{C}_+)$.

Finally, we have from [2, Lemma 2.5]

Lemma 4.2. *Let* $r \in \mathbf{R}$*. Then*

$$t_1^r, t_2^r, \omega_\lambda^r, (t_2 + t_1)^r \in \mathbb{M}_{r,1}(\mathbf{C}_+).$$

4.3. Proof of Theorem 2.1

Let us construct $\mathcal{R}$-bounded solutions operator families associated with $\mathcal{A}(\lambda)$, $\mathcal{B}_j(\lambda)$ $(j = 1, \dots, N-1)$, and $\mathcal{B}_N(\lambda)$ given in Subsection 4.1. To this end, we use [2, Lemmas 2.6 and 2.7] for the terms with g, while we use the following two lemmas for the terms with $h_1, \dots, h_{N-1}$.

Lemma 4.3. *Let* $q \in (1, \infty)$*. Suppose that*

$$k(\xi', \lambda) \in \mathbb{M}_{-2,1}(\mathbf{C}_+), \quad l(\xi', \lambda) \in \mathbb{M}_{-1,1}(\mathbf{C}_+),$$

and set for $x = (x', x_N) \in \mathbf{R}^N_+$

$$[K_0(\lambda)f](x) \; = \mathcal{F}^{-1}_{\xi'}\left[k(\xi', \lambda)e^{-\omega_\lambda x_N}\widehat{f}(\xi', 0)\right](x'),$$

with $\lambda \in \mathbf{C}_+$ *and* $f \in H^1_q(\mathbf{R}^N_+)$*. Then the following assertions hold.*

(1) For $j = 0, 1, 2$ *and* $\lambda \in \mathbf{C}_+$*, there exist operators* $\widetilde{K}_j(\lambda)$*, with*

$$\widetilde{K}_j(\lambda) \in Hol(\mathbf{C}_+, \mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1}, H^3_q(\mathbf{R}^N_+))),$$

such that for any $f \in H^1_q(\mathbf{R}^N_+)$

$$K_j(\lambda)f = \widetilde{K}_j(\lambda)(\nabla f, \lambda^{1/2} f).$$

In addition, for $j = 0, 1, 2$ *and* $n = 0, 1$,

$$\mathcal{R}_{\mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1}, \mathfrak{A}_q(\mathbf{R}^N_+))}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n \left(\mathcal{S}_\lambda \widetilde{K}_j(\lambda)\right) \middle| \lambda \in \mathbf{C}_+\right\}\right) \leq C,$$

with some positive constant $C = C(N, q, \mu, \nu, \kappa)$.

(2) For $j = 0, 1, 2$ *and* $\lambda \in \mathbf{C}_+$*, there exist operators* $\widetilde{L}_j(\lambda)$*, with*

$$\widetilde{L}_j(\lambda) \in Hol(\mathbf{C}_+, \mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1}, H^2_q(\mathbf{R}^N_+))),$$

such that for any $f \in H^1_q(\mathbf{R}^N_+)$

$$L_j(\lambda)f = \widetilde{L}_j(\lambda)(\nabla f, \lambda^{1/2} f).$$

In addition, for $j = 0, 1, 2$ *and* $n = 0, 1$,

$$\mathcal{R}_{\mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1}, L_q(\mathbf{R}^N_+)^{N^2+N+1})}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n \left(\mathcal{T}_\lambda \widetilde{L}_j(\lambda)\right) \middle| \lambda \in \mathbf{C}_+\right\}\right) \leq C,$$

with some positive constant $C = C(N, q, \mu, \nu, \kappa)$.

Lemma 4.4. *Let* $q \in (1,\infty)$. *Suppose that*

$$m_0(\xi',\lambda) \in \mathbb{M}_{-1,1}(\mathbf{C}_+), \quad m_1(\xi',\lambda), m_2(\xi',\lambda) \in \mathbb{M}_{0,1}(\mathbf{C}_+),$$

and set for $x = (x', x_N) \in \mathbf{R}^N_+$

$$[M_k(\lambda)f](x) = \mathcal{F}^{-1}_{\xi'}\left[m_k(\xi',\lambda)\mathcal{M}_k(x_N)\widehat{f}(\xi',0)\right](x'),$$

with $\lambda \in \mathbf{C}_+$ *and* $f \in H^1_q(\mathbf{R}^N_+)$. *Then the following assertions hold.*

(1) *For* $\lambda \in \mathbf{C}_+$, *there exists an operator* $\widetilde{M}_0(\lambda)$, *with*

$$\widetilde{M}_0(\lambda) \in Hol(\mathbf{C}_+, L_q(\mathbf{R}^N_+)^{N+1}, H^3_q(\mathbf{R}^N_+)),$$

such that for any $f \in H^1_q(\mathbf{R}^N_+)$

$$M_0(\lambda)f = \widetilde{M}_0(\lambda)(\nabla f, \lambda^{1/2}f).$$

In addition, for $n = 0, 1$,

$$\mathcal{R}_{\mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1},\mathfrak{A}_q(\mathbf{R}^N_+))}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n\left(\mathcal{S}_\lambda\widetilde{M}_0(\lambda)\right)\Big|\,\lambda \in \mathbf{C}_+\right\}\right) \leq C,$$

with some positive constant $C = C(N,q,\mu,\nu,\kappa)$.

(2) *For* $j = 1, 2$ *and* $\lambda \in \mathbf{C}_+$, *there exists operators* $\widetilde{M}_j(\lambda)$, *with*

$$\widetilde{M}_j(\lambda) \in Hol(\mathbf{C}_+, \mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1}, H^2_q(\mathbf{R}^N_+))),$$

such that for any $f \in H^1_q(\mathbf{R}^N_+)$

$$M_j(\lambda)f = \widetilde{M}_j(\lambda)(\nabla f, \lambda^{1/2}f).$$

In addition, for $j = 1, 2$ *and* $n = 0, 1$,

$$\mathcal{R}_{\mathcal{L}(L_q(\mathbf{R}^N_+)^{N+1},L_q(\mathbf{R}^N_+)^{N^2+N+1})}\left(\left\{\left(\lambda\frac{d}{d\lambda}\right)^n\left(\mathcal{T}_\lambda\widetilde{M}_j(\lambda)\right)\Big|\,\lambda \in \mathbf{C}_+\right\}\right) \leq C,$$

with some positive constant $C = C(N,q,\mu,\nu,\kappa)$.

By Lemmas 4.1 and 4.2, we observe that the symbols appearing the solution operators satisfy the following conditions: In the case of $\mathcal{A}(\lambda)$,

$$\begin{aligned}\frac{\lambda}{t_2(t_2+t_1)} &\in \mathbb{M}_{0,1}(\mathbf{C}_+),\\ \frac{i\xi_k}{t_2(t_2+t_1)}, \frac{1}{t_1}, \frac{1}{t_2} &\in \mathbb{M}_{-1,1}(\mathbf{C}_+),\\ \frac{i\xi_k}{t_1t_2(t_2+t_1)} &\in \mathbb{M}_{-2,1}(\mathbf{C}_+);\end{aligned}$$

In the case of $\mathcal{B}_j(\lambda)$ $(j = 1, \ldots, N-1)$,

$$\begin{aligned}\frac{1}{\omega_\lambda} \in \mathbb{M}_{-1,1}(\mathbf{C}_+), \quad \frac{i\xi_j(t_1-\omega_\lambda)}{t_1\omega_\lambda}, \frac{i\xi_j(t_2-\omega_\lambda)}{t_2\omega_\lambda} &\in \mathbb{M}_{0,1}(\mathbf{C}_+),\\ \frac{\xi_j\xi_k}{t_1\omega_\lambda(t_1+\omega_\lambda)}, \frac{\xi_j\xi_k}{t_2\omega_\lambda(t_2+\omega_\lambda)} &\in \mathbb{M}_{-1,1}(\mathbf{C}_+),\\ \frac{i\xi_j\lambda}{t_1(t_2+t_1)}, \frac{i\xi_j\lambda}{t_2(t_2+t_1)} &\in \mathbb{M}_{1,1}(\mathbf{C}_+),\\ \frac{\xi_j\xi_k}{t_1(t_2+t_1)}, \frac{\xi_j\xi_k}{t_2(t_2+t_1)} &\in \mathbb{M}_{0,1}(\mathbf{C}_+);\end{aligned}$$

In the case of $\mathcal{B}_N(\lambda)$,

$$\frac{\lambda}{t_2+t_1} \in \mathbb{M}_{1,1}(\mathbf{C}_+), \quad \frac{i\xi_k}{t_2+t_1} \in \mathbb{M}_{0,1}(\mathbf{C}_+).$$

Thus, applying [2, Lemmas 2.6 and 2.7] and Lemmas 4.3 and 4.4 to $\mathcal{A}(\lambda)$, $\mathcal{B}_j(\lambda)$ $(j = 1, \ldots, N-1)$, and $\mathcal{B}_N(\lambda)$, we obtain the $\mathcal{R}$-bounded solution operator families stated in Theorem 2.1. This completes the proof of Theorem 2.1

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