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Notification of Acceptance (MATH_07)

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ASMIATI 1976 <asmiati.1976@fmipa.unila.ac.id>

to me

Wed, Sep 9, 2020, 8:39 AM

Dear **Sri Maryani,**

We are happy to inform you that your paper entitled "**Solution Formula of the Compressible Fluid Motion in Three Dimension Euclidean Space using Fourier Transform**" is accepted with **major revisions** to publish in IOP Proceedings. Please revise the paper according to the reviewer comments and ensure the format matches ICASMI template that we sent in the attachment. Hopefully we will receive the revised paper **on September 14, 2020**.

Thank you for your attention and cooperation.

Best regards,

Dr. Asmiat

3 Attachments

- MATH_07.pdf
- Review_MATH_7.pdf
- Template-ICASMI ...

Solution Formula of the Compressible Fluid Motion in Three Dimension Euclidean Space using Fourier Transform

Abstract. We derive a detailed determination of the solution formula for the compressible viscous fluid flow in three dimensional Euclidean space using Fourier transform. For the further research, we can not only generalized the model problem to the N-dimensional Euclidean space (N>3) but also we can estimate the solution operator families of the model problem.

Keyword: Compressible, Euclidean space, Fourier transform, Viscous fluid,.

1. Introduction

In this paper, we consider the solution formula of the linearized for compressible fluid flow which described as follows:

$$\begin{cases} \rho_t + \gamma div\, \mathbf{u} = 0 \\ \mathbf{v}_t - \mu\Delta\mathbf{u} - \nu\nabla div\, \mathbf{u} - \kappa\nabla\Delta\rho = \mathbf{0} \end{cases} \tag{1}$$

With the initial data are $\mathbf{u}|_{\partial\Omega} = 0$, $(\rho, \mathbf{u})|_{t=0} = (\rho_0, \mathbf{u}_0)$ and $\mathbf{u} = \langle u_1, u_2, u_3 \rangle^T$ \$ is velocity and $\langle u_1, u_2, u_3 \rangle^T$ is the transpose of $\langle u_1, u_2, u_3 \rangle$. We know that $\rho = \rho(x)$ and $\mathbf{u} = \langle u_1(x), u_2(x), u_3(x) \rangle^T$ are respectively the fluid density and the fluid velocity that are unknown functions. Whilts, μ and γ are positive constants, and ν is a constant such that $\mu + \nu > 0$ and μ and ν are the viscosity coefficients. The domain Ω is a three dimensional Euclidean space $\mathbb{R}^3$. This result can be generalized to $\mathbb{N}$-dimensional case and also we can estimates the solution operator families of the model problem which investigated by [2] in 2016.

To introduce our main result, we introduce the notation. For a scalar-valued function $u = u(x)$ and vector-valued function $\mathbf{v} = \mathbf{v}(x) = \langle v_1(x), v_2(x), v_3(x) \rangle^T$, we set for $\partial_k = \frac{\partial}{\partial x_k}$, $(k = 1, \cdots, N)$

$$\nabla u = (\partial_1 u, \partial_2 u, \partial_3 u)^T, \quad \Delta u = \textstyle\sum_{k=1}^3 \partial_k^2 u, \quad \Delta\mathbf{v} = \langle \Delta v_1, \Delta v_2, \Delta v_3 \rangle^T,$$

$$\text{div } \mathrm{v} = \textstyle\sum_{k=1}^3 \partial_k v_k, \quad \nabla\mathbf{v} = \{\partial_k v_l \mid \mathrm{k,l} = 1,2,3\}, \quad \nabla^2\mathbf{v} = \{\partial_k\partial_l v_m \mid k,l,m = 1,2,3\}.$$

The set of all natural number is denoted by $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We set,

$$W_p^{k,m}(\Omega) = \{\mathbb{U} = (\rho, \mathbf{v}) \mid \rho \in W_p^k(\Omega), \mathbf{v} \in W_p^m(\Omega)\}.$$

Before we state the main result, first of all we introduce the definition of Sobolev space $W_q^m(\Omega)$.

Definition 1.1 (Adams and Fournier, [1])
Let $k \in \mathbb{N} \cup \{0\}$ and $p \in [1, \infty)$ then the Sobolev Space $W_q^m(\Omega)$ is defined by

$$W_q^m(\Omega) := \{u \in L_q(\Omega) \mid D^\alpha u \in L_q(\Omega), \forall\, \alpha \text{ with } |\alpha| \le m\}$$

Next, we state the main result of this paper.

Theorem 1.2 Let $\{\lambda_j(\xi)\}_{j=1}^{4}$ be the roots of $\det[\lambda\mathbb{I} + \widehat{\mathbb{M}}(\lambda)] = 0$, where $\lambda_3(\xi) = -\alpha|\xi|^2.$
Then for $\lambda_j(\xi), j = 1,2$, we have the following assertions:

i. For $|\xi| \ge \frac{2\gamma}{(\alpha+\beta)}$, we have $\lambda_j(\xi), j = 1,2$ as follows:

$$\lambda_1 = \frac{-(\alpha+\beta)}{2}|\xi|^2 + \frac{1}{2}|\xi|\sqrt{(\alpha+\beta)^2|\xi|^2 - 4\gamma^2}$$

$$\lambda_2 = \frac{-(\alpha+\beta)}{2}|\xi|^2 - \frac{1}{2}|\xi|\sqrt{(\alpha+\beta)^2|\xi|^2 - 4\gamma^2}$$

ii. For $|\xi| \le \frac{2\gamma}{(\alpha+\beta)}$, we have $\lambda_j(\xi), j = 1,2$ as follows:

$$\lambda_1 = \overline{\lambda_2} = \frac{-(\alpha+\beta)}{2}|\xi|^2 + \frac{i}{2}|\xi|\sqrt{4\gamma^2 - (\alpha+\beta)^2|\xi|^2},\quad i = \sqrt{-1}$$

iii. For $|\xi| \ne \frac{2\gamma}{(\alpha+\beta)}$, we have the solution formula of $\hat\rho(\xi,t)$ and $\hat{\mathbf{v}}(\xi,t)$ as follow

$$\hat\rho(\xi,t) = \left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)t} - \lambda_1(\xi)e^{\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat\rho_0(\xi,t) - i\left(\frac{e^{\lambda_2(\xi)t} - e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi),$$

$$\hat{\mathbf{v}}(\xi,t) = i\gamma\xi\left(\frac{e^{\lambda_2(\xi)t} - e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat\rho_0(\xi,t) + e^{-\alpha|\xi|^2 t}\hat{\mathbf{v}}_0(\xi)$$
$$+\left(\frac{\lambda_2(\xi)e^{\lambda_2(\xi)t} - \lambda_1(\xi)e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)} - e^{-\alpha|\xi|^2 t}\right)\frac{\xi\xi^T\hat{\mathbf{v}}_0(\xi)}{|\xi|^2}.$$

This paper is organized as follows: the next section introduce a reduced system for (1.1) and shows that Theorem 1.2 follows from the main result for the reduced system. In third Section, we calculate representation formulas for solutions of the reduced system by using the Fourier transform with respect to $x = (x_1, x_2, x_3)$ and its inverse transform. Section 4 proves our main theorem for the reduced system by results obtained in Section 3.

2. Reduced system

In this section, we consider the resolvent problem of eaution system (1). Set $u_j = v_j$ $(j = 1,2,3)$, then $\mathbf{v} = \langle v_1, v_2, v_3\rangle^T$ satisfies

$$\begin{cases} \lambda\rho + \gamma \operatorname{div}\mathbf{v} = \tilde f & \text{in } \Omega, \\ \lambda\mathbf{v} - \mu\Delta - \nu\nabla\operatorname{div}\mathbf{v} - \kappa\nabla\Delta\rho = \tilde{\boldsymbol{g}} & \text{in } \Omega, \end{cases} \tag{2}$$

First of all, we can wrte the equation system (2) in the following

$$(\lambda + \mathbb{M})\mathbb{U} = \mathbb{F} \qquad \text{in } \Omega \tag{3}$$

where,

$$\mathbb{U} = \begin{pmatrix} \rho \\ v \end{pmatrix} = \begin{pmatrix} \rho \\ v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad \mathbb{M} = \begin{bmatrix} 0 & \gamma div \\ \gamma\nabla & -\alpha\Delta - \beta\nabla div \end{bmatrix}, \quad and \quad \mathbb{F} = \begin{pmatrix} \tilde{f} \\ \tilde{g} \end{pmatrix}.$$

We can also write the equation system of (2) in a matrix form

$$\lambda \begin{pmatrix} \rho \\ v \end{pmatrix} + \begin{bmatrix} 0 & \gamma div \\ \gamma\nabla & -\alpha\Delta - \beta\nabla div \end{bmatrix} \begin{pmatrix} \rho \\ v \end{pmatrix} = \begin{pmatrix} \tilde{f} \\ \tilde{g} \end{pmatrix},$$

check again in eq (2)

or we can write in the form

$$\lambda \begin{pmatrix} \rho \\ v \end{pmatrix} + \begin{bmatrix} 0 & \gamma \sum_{k=1}^{3} \partial_k \\ \gamma\nabla & -\alpha\partial_k^2 - \beta\partial_k \sum_{j=1}^{3} \partial_j \end{bmatrix} \begin{pmatrix} \rho \\ v \end{pmatrix} = \begin{pmatrix} \tilde{f} \\ \tilde{g} \end{pmatrix}.$$

In addition, for the time derivative, we can write the equation system of (1) in the following

$$\begin{cases} \rho_t + \gamma div\, \mathbf{v} = \mathbf{0} & in\ [0,\infty) \times \mathbb{R}^3, \\ \mathbf{v}_t - \mu\Delta\mathbf{v} - \nu\nabla div\, \mathbf{v} - \kappa\nabla\Delta\rho = \mathbf{0} & in\ [0,\infty) \times \mathbb{R}^3. \end{cases} \tag{3}$$

Furthermore, we consider the equation (3). For the simplicity, we can write the equation (3)

$$\mathbb{U}_t + \mathbb{M}\mathbb{U} = 0 \quad in\ [0,\infty) \times \mathbb{R}^3, \quad \mathbb{U}\mid_{t=0} = \mathbb{U}_0 \quad in\ \mathbb{R}^3, \tag{4}$$

with domain $D(\mathbb{M}) = \{ \mathbb{U} = (\rho, v) \in W_p^{1,2} \mid v \mid_{\partial\Omega} = 0 \}$, $\mathbb{U}_0 = (\rho_0, v_0)$. Then, by taking Fourier transform to (3) with respect to the x variable and solving the ordinary differential equation with respect to variable t, we have

$$\mathbb{U}_t = \mathbb{E}(t)\mathbb{F} = \mathcal{F}^{-1}\left(e^{-t\widehat{\mathbb{M}}(\xi)}\widehat{\mathbb{F}}(\xi)\right),$$

where we define the Fourier transform $\hat{u}$ of $u = u(x)$ with respect to $x = (x_1, x_2, x_3)$ and its inverse transform as follows:

$$\hat{u} = \hat{u}(\xi) = \hat{u}(x) = \int_{\mathbb{R}^3} e^{-ix\cdot\xi}\, u(x)dx$$

$$= \mathcal{F}^{-1}[\hat{u}(\xi)](x) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} e^{ix\cdot\xi}\, \hat{u}(\xi)d\xi$$

Where $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$.

Moreover, we also can apply Fourier transform with respect to variable x to equation (4),

$$\lambda\widehat{\mathbb{U}} + \widehat{\mathbb{M}}(\xi)\widehat{\mathbb{U}} = \widehat{\mathbb{F}}$$

$$[\lambda + \widehat{\mathbb{M}}(\xi)]\widehat{\mathbb{U}} = \widehat{\mathbb{F}}$$

$$\hat{\mathbb{U}} = [\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]^{-1}\hat{\mathbb{F}}(\xi)$$
$$\mathbb{U} = \mathcal{F}^{-1}\left\{[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]^{-1}\hat{\mathbb{F}}(\xi)\right\} \tag{5}$$

with $\det[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)] \neq 0$ and $[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]^{-1}$ is inverse of $[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]$.

Furthermore, by using adjoint of the matrix, we can determine the inverse of matrix,

$$[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]^{-1} = \frac{1}{\det[\lambda\mathbf{I}+\hat{\mathbb{M}}(\xi)]}\operatorname{adj}[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]. \tag{6}$$

3. Representation formulas for solutions

In this section, following [4, Section III], we compute representation formulas for solutions of (2). First of all, applying the Fourier transform to matrix $\mathbb{M}$ in equation (6) yield the following matrix

$$\hat{\mathbb{M}}(\xi) = \begin{bmatrix} 0 & i\gamma\xi_1 & i\gamma\xi_2 & i\gamma\xi_3 \\ i\gamma\xi_1 & \mu|\xi|^2 + \nu\xi_1^2 & \nu\xi_1\xi_2 & \nu\xi_1\xi_3 \\ i\gamma\xi_2 & \nu\xi_2\xi_1 & \mu|\xi|^2 + \nu\xi_2^2 & \nu\xi_2\xi_3 \\ i\gamma\xi_3 & \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \mu|\xi|^2 + \nu\xi_3^2 \end{bmatrix}$$

where $|\xi|^2 = \xi_1^2 + \xi_2^2 + \xi_3^2$ and $i^2 = -1$. Then, we also have

$$[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)] = \begin{bmatrix} \lambda & i\gamma\xi_1 & i\gamma\xi_2 & i\gamma\xi_3 \\ i\gamma\xi_1 & \lambda + \mu|\xi|^2 + \nu\xi_1^2 & \nu\xi_1\xi_2 & \nu\xi_1\xi_3 \\ i\gamma\xi_2 & \nu\xi_2\xi_1 & \lambda + \mu|\xi|^2 + \nu\xi_2^2 & \nu\xi_2\xi_3 \\ i\gamma\xi_3 & \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \lambda + \mu|\xi|^2 + \nu\xi_3^2 \end{bmatrix}.$$

Then, we calculate for the determinant of matrix $[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]$ using expansion by cofactors, that is

$$\begin{aligned}\det[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)] &= \hat{a}_{11}\hat{\mathbb{C}}_{11} + \hat{a}_{12}\hat{\mathbb{C}}_{12} + \hat{a}_{13}\hat{\mathbb{C}}_{13} + \hat{a}_{14}\hat{\mathbb{C}}_{14} \\ &= \lambda|\hat{\mathbb{A}}_{11}| - i\gamma\xi_1|\hat{\mathbb{A}}_{12}| + i\gamma\xi_2|\hat{\mathbb{A}}_{13}| - i\gamma\xi_3|\hat{\mathbb{A}}_{14}|\end{aligned} \tag{7}$$

where $|\hat{\mathbb{A}}_{ij}|$ is determinant of submatrix that remains after the i-th row and j-th column are deleted from matrix $[\lambda\mathbf{I} + \hat{\mathbb{M}}(\xi)]$ and the number $(-1)^{i+j}\hat{\mathbb{A}}_{ij}$ is denoted by $\mathbb{C}_{ij}$ and called the cofactor.

In fact, for $i=j=1$ the component of $|\hat{\mathbb{A}}_{ij}|$, we have

$$|\hat{\mathbb{A}}_{11}| = \begin{vmatrix} \lambda + \mu|\xi|^2 + \nu\xi_1^2 & \nu\xi_1\xi_2 & \nu\xi_1\xi_3 \\ \nu\xi_2\xi_1 & \lambda + \mu|\xi|^2 + \nu\xi_2^2 & \nu\xi_2\xi_3 \\ \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \lambda + \mu|\xi|^2 + \nu\xi_3^2 \end{vmatrix}$$

$$= (\lambda + \mu|\xi|^2)^2\{(\lambda + \mu|\xi|^2) + \nu|\xi|^2\}.$$

Similar technique, we have $(i\gamma\xi_1)(\lambda + \mu|\xi|^2)^2, -(i\gamma\xi_2)(\lambda + \mu|\xi|^2)^2$ and $(i\gamma\xi_3)(\lambda + \mu|\xi|^2)^2$ for $|\hat{\mathbb{A}}_{12}|$, $|\hat{\mathbb{A}}_{13}|$, and $|\hat{\mathbb{A}}_{14}|$, respectively. Substituting $|\hat{\mathbb{A}}_{11}|$, $|\hat{\mathbb{A}}_{12}|$, $|\hat{\mathbb{A}}_{13}|$, and $|\hat{\mathbb{A}}_{14}|$ to equation (7) we have,

$$\det[\lambda \mathbf{I} + \widehat{\mathbb{M}}(\xi)] = (\lambda + \mu|\xi|^2)^2\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} \quad (8)$$

Furthermore, we determine a matrix adjoint of equation (7) which is a tranpose matrix of cofactor matrix. Since these matrix is a symmetric matrix, so that the determinant of the matrix hold the properties $|\widehat{\mathbb{A}}_{ij}| = |\widehat{\mathbb{A}}_{ji}|$ for $i, j = 1, 2, 3, 4$. Moreover, we enough only determine the $|\widehat{\mathbb{A}}_{22}|$, $|\widehat{\mathbb{A}}_{23}|$, $|\widehat{\mathbb{A}}_{24}|$, $|\widehat{\mathbb{A}}_{33}|$, $|\widehat{\mathbb{A}}_{34}|$ and $|\widehat{\mathbb{A}}_{44}|$. In fact, for $i = j$ we have

$$|\widehat{\mathbb{A}}_{22}| = (\lambda + \mu|\xi|^2)^2\{\lambda(\lambda + \mu|\xi|^2) + (\xi_2^2 + \xi_3^2)(\lambda\nu + \gamma^2)\},$$

$$|\widehat{\mathbb{A}}_{33}| = (\lambda + \mu|\xi|^2)^2\{\lambda(\lambda + \mu|\xi|^2) + (\xi_1^2 + \xi_3^2)(\lambda\nu + \gamma^2)\},$$

$$|\widehat{\mathbb{A}}_{44}| = (\lambda + \mu|\xi|^2)\ \ \{\lambda(\lambda + \mu|\xi|^2) + (\xi_1^2 + \xi_2^2)(\lambda\nu + \gamma^2)\}.$$

Employing the same argument, we can find others minors

$$|\widehat{\mathbb{A}}_{23}| = (\lambda + \mu|\xi|^2)\{(\xi_1\xi_2)(\lambda\nu + \gamma^2)\} = |\widehat{\mathbb{A}}_{32}|,$$

$$|\widehat{\mathbb{A}}_{24}| = -(\lambda + \mu\,|\xi|^2)\{(\xi_1\xi_3)(\lambda\nu + \gamma^2)\} = |\widehat{\mathbb{A}}_{42}|,$$

$$|\widehat{\mathbb{A}}_{34}| = -(\lambda + \mu|\xi|^2)\{(\xi_2\xi_3)(\lambda\nu + \gamma^2)\} = |\widehat{\mathbb{A}}_{32}|.$$

Moreover, we have the cofactors in the following

$$\hat{\mathbb{C}}_{11} = (\lambda + \mu|\xi|^2)^2\{(\lambda + \mu|\xi|^2) + \nu|\xi|^2\},$$

$$\hat{\mathbb{C}}_{1j} = \hat{\mathbb{C}}_{j1} - (i\gamma\xi_{j-1})(\lambda + \mu|\xi|^2)^2,$$

$$\hat{\mathbb{C}}_{ij} = \hat{\mathbb{C}}_{ji}(\lambda + \mu|\xi|^2)\{\lambda(\lambda + \mu|\xi|^2)\delta_{ij} + (\delta_{ij} - \xi_{i-1}\xi_{j-1})(\lambda\nu + \gamma^2)\}$$

with $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ij} = 1$ for $i = j$ and $i, j = 2, 3, 4$.

4. Proof of Theorem 2.1

Throughout this section, we use the notation introduced in the previous section.

4.1 Eigen Values

In this subsection, we investigate the eigen values. The solution formula for the model problem has already obtained by Kobayashi [5]. Here in after, we shall give aslightly detai how to get the solution formula for velocity and density in equation system (1). First of all, we determine the detreminant of the matrix $[\lambda \mathbf{I} + \widehat{\mathbb{M}}(\xi)] = 0$. By equation (8) we have

$$\det[\lambda \mathbf{I} + \widehat{\mathbb{M}}(\xi)] = (\lambda + \mu|\xi|^2)^2\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|\text{^}2\}$$

$$0 = (\lambda + \mu|\xi|^2)^2\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|\text{^}2\}. \quad (9)$$

From equation (9), we have two possibilities zero values, that are $(\lambda + \mu|\xi|^2)^2 = 0$ or $\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|\text{^}2\} = 0$. For the first case, we have $\lambda_3 = \lambda_4 = -\mu|\xi|^2$. Furthermore, we will find the eigen values of $\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|\text{^}2\} = 0$. By using the formula

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

we have,

$$\lambda_{1,2} = \frac{-(\mu+\nu)|\xi|^2 \pm |\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}}{2}. \tag{10}$$

In view of equation (10), for $|\xi| \geq \frac{2\gamma}{(\mu+\nu)}$, we have

$$\lambda_1 = \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2},$$

$$\lambda_2 = \frac{-(\mu+\nu)|\xi|^2 \pm |\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}}{2},$$

Meanwhile, for $|\xi| \leq \frac{2\gamma}{(\mu+\nu)}$, *we have*

$$\lambda_1 = \overline{\lambda_2} = \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{i}{2}|\xi|\sqrt{4\gamma^2 - (\mu+\nu)^2|\xi|^2}.$$

how about if $|\xi| = \frac{2\gamma}{(\mu+\nu)}$

4.2 Fourier transform of density $\hat{\rho}$ and velocity $\hat{\boldsymbol{v}}$

In this subsection, we consider the formula of $\hat{\rho}$ and $\hat{\boldsymbol{v}}$ for density and velocity, respectively. First of all, applying div to second equation (3), we have

what it it mean ?

$$\frac{\partial}{\partial t}\operatorname{div}\mathbf{v} - \mu\Delta\boldsymbol{v} - \nu\nabla\operatorname{div}\boldsymbol{v} - \kappa\nabla\Delta\rho = \mathbf{0}. \tag{11}$$

Let $D = \operatorname{div}\mathbf{v}$, then equation (11) can be written as follows

$$D_t - \omega\Delta D + \gamma\Delta\rho = 0, \tag{12}$$

what is relation of κ and γ in eq (11) and (12)

with $\omega = \mu + \nu$ and $D_t = \frac{\partial D}{\partial t}$. Recalling first equation (3),

$$\rho_t = \gamma \operatorname{div}\mathbf{v},$$

→ $\rho_t = -\gamma \operatorname{div} v$

and we can write above equation to be

$$\rho_t = -\gamma D,$$

$$\frac{\rho_t}{\gamma} = -D. \tag{13}$$

Here in after, we differentiate (13) respect to t variable and then substitute (12) and (13) to the result, we have

$$\frac{\rho_{tt}}{\gamma} = -D_t,$$

$$\frac{1}{\gamma}\rho_{tt} = \gamma\Delta\rho - \omega\Delta D,$$

$$\frac{1}{\gamma}\rho_{tt} = \gamma\Delta\rho - \omega\Delta(-\frac{\rho_t}{\gamma}),$$

$$\rho_{tt} = \gamma^2\Delta\rho + \omega\Delta\rho_t. \tag{14}$$

Applying Fourier transformation to equation (14), we have

$$\hat{\rho}_{tt} + \omega|\xi|^2\hat{\rho}_t + \gamma^2|\xi|^2\hat{\rho} = 0, \tag{15}$$

with the initial condition

$$\hat{\rho}(\xi,0)=\hat{\rho}_0(\xi),\ \ \hat{\rho}_t(\xi,0)=-i\xi\,\hat{\mathbf{v}}_0(\xi).$$

Moreover, we have the general solution for equation (15)

$$\hat{\rho}(\xi,t)=c_1e^{\lambda_1(\xi)t}+c_2e^{\lambda_2(\xi)t}, \tag{16}$$

where

$$\lambda_{1,2}=\frac{-\omega|\xi|^2\pm|\xi|\sqrt{\omega^2|\xi|^2-4\gamma^2}}{2}.$$

Substituting the initial condition to (16), we obtain

$$c_1=\frac{\lambda_2(\xi)\hat{\rho}_0(\xi)+i\xi\,\hat{\mathbf{v}}_0(\xi)}{\lambda_2(\xi)-\lambda_1(\xi)},\quad c_2=\frac{\lambda_1(\xi)\hat{\rho}_0(\xi)+i\xi\,\hat{\mathbf{v}}_0(\xi)}{\lambda_2(\xi)-\lambda_1(\xi)}.$$

Therefore, we have the solution formula for $\hat{\rho}(\xi,t)$ in the following

$$\hat{\rho}(\xi,t)=\left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)t}-\lambda_1(\xi)e^{\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{\lambda_2(\xi)t}-e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi). \tag{17}$$

Furthermore, we determine solution formula for $\hat{\mathbf{v}}(\xi,t)$. Employing the sam argument [3, Section 3]. ? Firstly, applying Fourier transform to the second equation of (3), we have

$$\hat{\mathbf{v}}_t+\mu|\xi|^2\hat{\mathbf{v}}-i\mu\xi_j\textstyle\sum_{k=1}^{3}i\xi_k\hat{v}_k+i\gamma\xi_j\hat{\rho}=0, \tag{18}$$

we can write (18) in the following

$$\hat{\mathbf{v}}_t=(-\mu|\xi|^2\mathbf{I}-\nu\xi\xi^T)\hat{\mathbf{v}}-i\gamma\xi\hat{\rho}. \tag{19}$$

Vector $\hat{\mathbf{v}}(\xi,t)$ is a vector which parallel and orthogonal from ξ, so that we can write the vector $\hat{\mathbf{v}}(\xi,t)$ as follows

$$\hat{\mathbf{v}}(\xi,t)=a(\xi,t)\frac{\xi}{|\xi|}+b(\xi,t), \tag{20}$$

where $b(\xi,t)$ orthogonal to ξ, and $a(\xi,t)$ is a scalar such that $a(\xi,t)=\hat{\mathbf{v}}(\xi,t)\frac{\xi}{|\xi|}$.

Substituting equation (20) to (19), then we have

$$\begin{aligned}\hat{\mathbf{v}}_t&=(-\mu|\xi|^2\mathbf{I}-\nu\xi\xi^T)\left(a(\xi,t)\frac{\xi}{|\xi|}+b(\xi,t)\right)-i\gamma\xi\hat{\rho},\\&=-a(\xi,t)\frac{\xi}{|\xi|}(\mu|\xi|^2\mathbf{I}+\nu\xi\xi^T)-b(\xi,t)(\mu|\xi|^2\mathbf{I}+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}\\&=-a(\xi,t)\left(\mu|\xi|\xi+\nu\frac{\xi\cdot\xi\xi^T}{|\xi|}\right)-b(\xi,t)(\mu|\xi|^2+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}.\end{aligned} \tag{21}$$

Next, we differentiate equation (20) respect to variable t, then substitute the result to equation (21), we obtain

$$a_t(\xi,t)\frac{\xi}{|\xi|}+b_t(\xi,t)=-a(\xi,t)\left(\mu|\xi|\xi+\nu\frac{\xi\cdot\xi\xi^T}{|\xi|}\right)-b(\xi,t)(\mu|\xi|^2\mathbf{I}+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}. \tag{21}$$

Therefore, we have

$$a_t(\xi,t)=\omega|\xi|^2a(\xi,t)-i\gamma|\xi|\hat{\rho},\quad b_t(\xi,t)=-\mu|\xi|^2b(\xi,t). \tag{22}$$

By (22) and initial condition $b(\xi,0)=\left(\mathbf{I}-\frac{\xi\xi^T}{|\xi|^2}\right)\hat{\mathbf{v}}_0$, we see that

$$b(\xi,t)=e^{-\mu|\xi|^2 t}\left(\mathbf{I}-\frac{\xi\xi^T}{|\xi|^2}\right)\hat{\mathbf{v}}_0. \tag{23}$$

Also, by taking integrating factor to (23) we have

$$a(\xi,t)=e^{-\omega|\xi|^2 t}\left(a(\xi,0)-i\gamma|\xi|\int_0^t e^{\omega|\xi|^2 s}\hat{\rho}(\xi,s)\ ds\right), \tag{24}$$

with $a(\xi,0)$ is a constant.

Furthermore, we consider the formula of the second term of (24). Multiplying (17) by factor $e^{-\omega|\xi|^2 t}$ we have

$$\begin{aligned} e^{\omega|\xi|^2 s}\hat{\rho}(\xi,s) &= \left(\frac{\lambda_2(\xi)e^{\omega|\xi|^2 s+\lambda_1(\xi)t}-\lambda_1(\xi)e^{\omega|\xi|^2 s+\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi) \\ &\quad -i\left(\frac{e^{\omega|\xi|^2 s+\lambda_2(\xi)t}-e^{\omega|\xi|^2 s+\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi) \\ &= \left(\frac{\lambda_2(\xi)e^{-\lambda_2(\xi)s}-\lambda_1(\xi)e^{-\lambda_1(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{-\lambda_1(\xi)s}-e^{-\lambda_2(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \end{aligned} \tag{25}$$

since, $\lambda_{1,2}(\xi)+\omega|\xi|^2=-\lambda_{2,1}(\xi)$ and $\lambda_1(\xi)\lambda_2(\xi)=\gamma|\xi|\text{^}2.$ is it $\gamma|\xi|^2$ or what. ?

Furthermore, by integrating (25) form $0\le s\le t$, we have

$$\begin{aligned} \int_0^t e^{\omega|\xi|^2 s}\hat{\rho}(\xi,s)\ ds &= \int_0^t \left(\frac{\lambda_2(\xi)e^{-\lambda_2(\xi)s}-\lambda_1(\xi)e^{-\lambda_1(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{-\lambda_1(\xi)s}-e^{-\lambda_2(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi)ds, \\ &= \left(\frac{-e^{-\lambda_2(\xi)t}+e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)+\frac{i}{\gamma|\xi|^2}\left(\frac{\lambda_2(\xi)e^{-\lambda_1(\xi)t}-\lambda_1(\xi)e^{-\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi). \end{aligned}$$

Without loss of generality, we take $a(\xi,0)=0$, so $a(\xi,0)e^{-\omega|\xi|^2 t}=0$, thus we have

$$a(\xi,t)=-i\gamma|\xi|\left(\frac{-e^{-\lambda_2(\xi)t}+e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)+\left(\frac{\lambda_2(\xi)e^{-\lambda_1(\xi)t}-\lambda_1(\xi)e^{-\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\frac{\xi\hat{\mathbf{v}}_0(\xi)}{|\xi|} \tag{26}$$

Thus, combining (23) and (26) yields that the formula of velocity $\hat{\mathbf{v}}(\xi,t)$. This complete the proof of Theorem 1.2.

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Solution Formula of the Compressible Fluid Motion in Three Dimension Euclidean Space using Fourier Transform

Solution Formula of the Compressible Fluid Motion in Three Dimension Euclidean Space using Fourier Transform

A H Alif[1], S Maryani[2], S R Nurshiami[3]

[1,2,3] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Jenderal Soedirman University, Jl. Dr. Soeparno No 61, Purwokerto, Indonesia

email: abiyyualif7@gmail.com[1], sri.maryani@unsoed.ac.id[2], siti.nurshiami@unsoed.ac.id[3]

Abstract. We derive a detailed determination of the solution formula for the compressible viscous fluid flow in three dimensional Euclidean space using Fourier transform. For the further research, we can not only generalized the model problem to the N-dimensional Euclidean space (N>3) but also we can estimate the solution operator families of the model problem.

Keywords: compressible, viscous fluid, Euclidean space, Fourier transform.

1. Introduction

In this paper, we consider the solution formula of the linearized for compressible fluid flow which described as follows:

$$\begin{cases} \rho_t + \gamma \text{div}\, \mathbf{v} = 0 \\ \mathbf{v_t} - \mu\Delta\mathbf{v} - \nu\nabla\text{div}\, \mathbf{v} + \gamma\nabla\rho = 0. \end{cases}, \tag{1}$$

with the initial data are $v|_{\partial\Omega} = 0$, $(\rho, \mathbf{v})|_{t=0} = (\rho_0, v_0)$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle^T$ is velocity and $\langle v_1, v_2, v_3 \rangle^T$ is the transpose of $\langle v_1, v_2, v_3 \rangle$. We know that density and velocity are written as $\rho = \rho(x)$ and $\mathbf{v} = \langle v_1(x), v_2(x), v_3(x) \rangle^T$, respectively. Moreover, μ and γ are positive constants, and ν is a constant such that $\mu + \nu > 0$ and μ and ν are the viscosity coefficients. The domain Ω is a three dimensional Euclidean space $\mathbb{R}^3$. This result can be generalized to $\mathbb{N}$-dimensional case and also we can estimates the solution operator families of the model problem which investigated by [2] in 2016.

To introduce our main result, first of all we introduce the notation. For a scalar-valued function $u = u(x)$ and vector-valued function $\mathbf{v} = \mathbf{v}(x) = \langle v_1(x), v_2(x), v_3(x)\rangle^T$, we set for $\partial_k = \frac{\partial}{\partial x_k}$, $(k = 1, \cdots, N)$

$$\nabla u = (\partial_1 u, \partial_2 u, \partial_3 u)^T, \quad \Delta u = \sum_{k=1}^3 \partial_k^2 u, \quad \Delta\mathbf{v} = \langle \Delta v_1, \Delta v_2, \Delta v_3\rangle^T,$$

$$\text{div}\, \mathbf{v} = \sum_{k=1}^3 \partial_k v_k, \quad \nabla v = \{\partial_k v_\ell \mid k, \ell = 1,2,3\}, \quad \nabla^2\mathbf{v} = \{\partial_k \partial_\ell v_m \mid k, \ell, m = 1, 2, 3\}.$$

The set of all natural number is denoted by $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We set,

$$W_p^{k,m}(\Omega) = \{\mathbb{U} = (\rho, \mathbf{v}) \mid \rho \in W_p^k(\Omega), \mathbf{v} \in W_p^m(\Omega)\}.$$

Before we state the main result, first of all we introduce the definition of Sobolev space $W_q^m(\Omega)$.

Definition 1.1 (Adams and Fournier, [1])
Let $k \in \mathbb{N} \cup \{0\}$ and $p \in [1, \infty)$ then the Sobolev Space $W_q^m(\Omega)$ is defined by

$$W_q^m(\Omega) := \{\mathbf{u} \in L_q(\Omega) \mid D^\alpha \mathbf{u} \in L_q(\Omega), \forall\, \alpha \text{ with } |\alpha| \le m\}$$

Next, we state the main result of this paper.

Theorem 1.2 Let $\lambda_j(\xi), j = 1, \dots, 4$ be the roots of $\det[\lambda \mathbb{I} + \widehat{\mathbb{M}}(\lambda)] = 0$, where $\lambda_3(\xi) = \lambda_4(\xi) = -\mu|\xi|^2$. Then for $\lambda_j(\xi), j = 1,2$, we have the following assertions:

i. For $|\xi| \ge \frac{2\gamma}{(\mu+\nu)}$, we have $\lambda_j(\xi), j = 1,2$ as follows:

$$\lambda_1 = \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}$$
$$\lambda_2 = \frac{-(\mu+\nu)}{2}|\xi|^2 - \frac{1}{2}|\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}.$$

ii. For $|\xi| \le \frac{2\gamma}{(\mu+\nu)}$, we have $\lambda_j(\xi), j = 1,2$ as follows:

$$\lambda_1 = \overline{\lambda_2}\ \frac{-(\mu+\nu)}{2}|\xi|^2 + \frac{i}{2}|\xi|\sqrt{4\gamma^2 - (\mu+\nu)^2|\xi|^2}, \quad i = \sqrt{-1}.$$

iii. For $|\xi| \ne \frac{2\gamma}{(\alpha+\beta)}$, we have the solution formula of $\hat{\rho}(\xi, t)$ and $\hat{\mathbf{v}}(\xi, t)$ as follow

$$\hat{\rho}(\xi,t) = \left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)t} - \lambda_1(\xi)e^{\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi,t) - i\left(\frac{e^{\lambda_2(\xi)t} - e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi),$$

$$\hat{\mathbf{v}}(\xi,t) = i\gamma\xi\left(\frac{e^{\lambda_2(\xi)t} - e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi,t) + e^{-\mu|\xi|^2 t}\hat{\mathbf{v}}_0(\xi)$$
$$+\left(\frac{\lambda_2(\xi)e^{\lambda_2(\xi)t} - \lambda_1(\xi)e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)} - e^{-\alpha|\xi|^2 t}\right)\frac{\xi\xi^T\hat{\mathbf{v}}_0(\xi)}{|\xi|^2}.$$

This paper is organized as follows: the next section introduce a reduced system for (1) and shows that **Theorem 1.2** follows from the main result for the reduced system. In third Section, we calculate representation formulas for solutions of the reduced system by using the Fourier transform with respect to $\mathbf{x} = \langle x_1, x_2, x_3\rangle$ and its inverse transform. Section 4 proves our main theorem for the reduced system by results obtained in Section 3.

2. Reduced System

In this section, we consider the resolvent problem of eqaution system (1). Set $u_j = v_j$ (j = 1,2,3), then $\mathbf{v} = \langle v_1, v_2, v_3\rangle^T$ satisfies

$$\begin{cases} \lambda\rho + \gamma \operatorname{div} \mathbf{v} = \tilde{f} & \text{in } \Omega, \\ \lambda\, \mathbf{v} - \mu\Delta\mathbf{v} - \nu\nabla \operatorname{div} \mathbf{v} + \gamma\nabla\rho = \tilde{\mathbf{g}} & \text{in } \Omega. \end{cases} \tag{2}$$

First of all, we can write the equation system of (2) in the following

$$(\lambda + \mathbb{M})\mathbb{U} = \mathbb{F} \qquad in\ \Omega \tag{3}$$

where,

$$\mathbb{U} = \begin{pmatrix} \rho \\ \boldsymbol{v} \end{pmatrix} = \begin{pmatrix} \rho \\ v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad \mathbb{M} = \begin{bmatrix} 0 & \gamma div \\ \gamma\nabla & -\mu\Delta - \nu\nabla div \end{bmatrix}, \ and \quad \mathbb{F} = \begin{pmatrix} \tilde{f} \\ \tilde{\boldsymbol{g}} \end{pmatrix}.$$

We can also write the equation system of (2) in a matrix form

$$\lambda\begin{pmatrix} \rho \\ \boldsymbol{v} \end{pmatrix} + \begin{bmatrix} 0 & \gamma div \\ \gamma\nabla & -\mu\Delta - \nu\nabla div \end{bmatrix}\begin{pmatrix} \rho \\ \boldsymbol{v} \end{pmatrix} = \begin{pmatrix} \tilde{f} \\ \tilde{\boldsymbol{g}} \end{pmatrix},$$

or we can write in the form

$$\lambda\binom{\rho}{\boldsymbol{v}}+\begin{bmatrix}0 & \gamma\sum_{k=1}^{3}\partial_k\\ \gamma\nabla & -\mu\partial_k^2-\nu\partial_k\sum_{j=1}^{3}\partial_j\end{bmatrix}\binom{\rho}{\boldsymbol{v}}=\binom{\tilde{f}}{\tilde{\boldsymbol{g}}}. \tag{4}$$

In addition, for the time derivative, we can write the equation system of (1) in the following

$$\begin{cases}\rho_t+\gamma div\,\boldsymbol{v}=\;0 & in\;[0,\infty)\times\mathbb{R}^3,\\ v_t-\mu\Delta\boldsymbol{v}-\nu\nabla div\,\boldsymbol{v}+\gamma\nabla\rho=0 & in\;[0,\infty)\times\mathbb{R}^3.\end{cases} \tag{5}$$

Furthermore, we consider the equation (5). For the simplicity, we can write the equation (5) to be

$$\mathbb{U}_t+\mathbb{M}\mathbb{U}=0\quad in\;[0,\infty)\times\mathbb{R}^3,\;\;\mathbb{U}\mid_{t=0}=\mathbb{U}_0\quad in\;\mathbb{R}^3, \tag{6}$$

with domain $D(\mathbb{M})=\{\mathbb{U}=(\rho,\boldsymbol{v})\in W_p^{1,2}\mid\boldsymbol{v}\mid_{\partial\Omega}=0\}$, $\mathbb{U}_0=(\rho_0,\boldsymbol{v}_0)$. Then, by taking Fourier transform to (6) with respect to the x variable and solving the ordinary differential equation with respect to variable t, we have

$$\mathbb{U}_t=\mathbb{E}(t)\mathbb{F}=\mathcal{F}^{-1}\left(e^{-t\widehat{\mathbb{M}}(\xi)}\widehat{\mathbb{F}}(\xi)\right),$$

where we define the Fourier transform $\hat{f}$of $f=f(x)$ with respect to $\boldsymbol{x}=\langle x_1,x_2,x_3\rangle$ and its inverse transform as follows:

$$\hat{f}\;=\mathcal{F}_x[f]\,(\xi)=\hat{u}\,(x)=\int_{\mathbb{R}^3}e^{-ix\cdot\xi}\,f(x)dx$$

$$\mathcal{F}_\xi^{-1}[g](x)=\frac{1}{(2\pi)^3}\int_{\mathbb{R}^3}e^{ix\cdot\xi}\,g(\xi)d\xi$$

where $\xi=\langle\xi_1,\xi_2,\xi_3\rangle\in\mathbb{R}^3$.

Next, we consider the resolvent problem of equation system (5) then applying Fourier transform, we have

$$\lambda\widehat{\mathbb{U}}+\widehat{\mathbb{M}}(\xi)\widehat{\mathbb{U}}=\widehat{\mathbb{F}}$$

$$[\lambda+\widehat{\mathbb{M}}(\xi)]\widehat{\mathbb{U}}=\widehat{\mathbb{F}}$$
$$\widehat{\mathbb{U}}=\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]^{-1}\widehat{\mathbb{F}}(\xi)$$
$$\mathbb{U}=\mathcal{F}^{-1}\left\{\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]^{-1}\widehat{\mathbb{F}}(\xi)\right\} \tag{7}$$

with $\det\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]\neq0$ and $\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]^{-1}$ is inverse of $\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]$.
Furthermore, by using adjoint of the matrix, we can determine the inverse of matrix $\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]$,

$$\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]^{-1}=\frac{1}{\det\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]}\mathrm{adj}\left[\lambda\mathrm{I}+\widehat{\mathbb{M}}(\xi)\right]. \tag{8}$$

3. Representation formulas for solution

In this section, following [4, section III], we compute representation formulas for solutions of (2). First of all, applying the Fourier transform to matrix $\mathbb{M}$ in equation (3) yield the following matrix

$$\widehat{\mathbb{M}}(\boldsymbol{\xi})=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{i\gamma\xi_1} & \boldsymbol{i\gamma\xi_2} & \boldsymbol{i\gamma\xi_3}\\ \boldsymbol{i\gamma\xi_1} & \boldsymbol{\mu|\xi|^2+\nu\xi_1^2} & \boldsymbol{\nu\xi_1\xi_2} & \boldsymbol{\nu\xi_1\xi_3}\\ \boldsymbol{i\gamma\xi_2} & \boldsymbol{\nu\xi_2\xi_1} & \boldsymbol{\mu|\xi|^2+\nu\xi_2^2} & \boldsymbol{\nu\xi_2\xi_3}\\ \boldsymbol{i\gamma\xi_3} & \boldsymbol{\nu\xi_3\xi_1} & \boldsymbol{\nu\xi_3\xi_2} & \boldsymbol{\mu|\xi|^2+\nu\xi_3^2}\end{bmatrix}$$

where $|\boldsymbol{\xi}|^2=\boldsymbol{\xi_1^2}+\boldsymbol{\xi_2^2}+\boldsymbol{\xi_3^2}$ and $\boldsymbol{i^2=-1}$. Then we also have

$$[\boldsymbol{\lambda I} + \widehat{\mathbb{M}}(\boldsymbol{\xi})] = \begin{bmatrix} \lambda & i\gamma\xi_1 & i\gamma\xi_2 & i\gamma\xi_3 \\ i\gamma\xi_1 & \lambda+\mu|\xi|^2+\nu\xi_1^2 & \nu\xi_1\xi_2 & \nu\xi_1\xi_3 \\ i\gamma\xi_2 & \nu\xi_2\xi_1 & \lambda+\mu|\xi|^2+\nu\xi_2^2 & \nu\xi_2\xi_3 \\ i\gamma\xi_3 & \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \lambda+\mu|\xi|^2+\nu\xi_3^2 \end{bmatrix}.$$

Then, we calculate for the determinant of matrix $[\lambda I + \widehat{\mathbb{M}}(\xi)]$ using expansion by cofactors, that is

$$\begin{aligned}\det[\lambda I + \widehat{\mathbb{M}}(\xi)] &= \hat{a}_{11}\widehat{C}_{11} + \hat{a}_{12}\widehat{C}_{12} + \hat{a}_{13}\widehat{C}_{13} + \hat{a}_{14}\widehat{C}_{14} \\ &= \lambda|\widehat{\mathbb{A}}_{11}| - i\gamma\xi_1|\widehat{\mathbb{A}}_{12}| + i\gamma\xi_2|\widehat{\mathbb{A}}_{13}| - i\gamma\xi_3|\widehat{\mathbb{A}}_{14}| \end{aligned} \tag{9}$$

where $|\widehat{\mathbb{A}}_{ij}|$ is determinant of submatrix that remains after the i-th row and j-th column are deleted from matrix $[\lambda I + \widehat{\mathbb{M}}(\xi)]$ and the number $(-1)^{i+j}\widehat{\mathbb{A}}_{ij}$ is denoted by $\widehat{C}_{ij}$ and called the cofactor.
In fact, for $i = j = 1$ the component of $|\widehat{\mathbb{A}}_{ij}|$, we have

$$\begin{aligned}|\widehat{\mathbb{A}}_{11}| &= \begin{vmatrix} \lambda+\mu|\xi|^2+\nu\xi_1^2 & \nu\xi_1\xi_2 & \beta\xi_1\xi_3 \\ \nu\xi_2\xi_1 & \lambda+\mu|\xi|^2+\nu\xi_2^2 & \nu\xi_2\xi_3 \\ \nu\xi_3\xi_1 & \nu\xi_3\xi_2 & \lambda+\mu|\xi|^2+\nu\xi_3^2 \end{vmatrix} \\ &= (\lambda+\mu|\xi|^2)^2\{(\lambda+\mu|\xi|^2)+\nu|\xi|^2\}.\end{aligned}$$

Similar technique, we have $(i\gamma\xi_1)(\lambda+\mu|\xi|^2)^2, -(i\gamma\xi_2)(\lambda+\mu|\xi|^2)^2$ and $(i\gamma\xi_3)(\lambda+\mu|\xi|^2)^2$ for $|\widehat{\mathbb{A}}_{12}|$, $|\widehat{\mathbb{A}}_{13}|$, and $|\widehat{\mathbb{A}}_{14}|$, respectively. Substituting $|\widehat{\mathbb{A}}_{11}|, |\widehat{\mathbb{A}}_{12}|$, $|\widehat{\mathbb{A}}_{13}|$, and $|\widehat{\mathbb{A}}_{14}|$ to equation (9) we have,

$$\det[\lambda I + \widehat{\mathbb{M}}(\xi)] = (\lambda+\alpha|\xi|^2)^2\{\lambda^2 + (\mu+\nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} \tag{10}$$

Furthermore, we determine a matrix adjoint of $[\lambda I + \widehat{\mathbb{M}}(\xi)]$ which is a tranpose matrix of cofactor matrix. Since these matrix is a symmetric matrix, so that the determinant of the matrix hold the properties $|\widehat{\mathbb{A}}_{ij}| = |\widehat{\mathbb{A}}_{ji}|$ for $i, j = 1, 2, 3, 4$. Moreover, we enough only determine the $|\widehat{\mathbb{A}}_{22}|$, $|\widehat{\mathbb{A}}_{23}|$, $|\widehat{\mathbb{A}}_{24}|$, $|\widehat{\mathbb{A}}_{33}|$, $|\widehat{\mathbb{A}}_{34}|$ and $|\widehat{\mathbb{A}}_{44}|$. In fact, for $i = j$ we have

$$\begin{aligned}|\widehat{\mathbb{A}}_{22}| &= (\lambda+\mu|\xi|^2)^2\{\lambda(\lambda+\mu|\xi|^2) + (\xi_2^2+\xi_3^2)(\lambda\nu+\gamma^2)\}, \\ |\widehat{\mathbb{A}}_{33}| &= (\lambda+\mu|\xi|^2)^2\{\lambda(\lambda+\mu|\xi|^2) + (\xi_1^2+\xi_3^2)(\lambda\nu+\gamma^2)\}, \\ |\widehat{\mathbb{A}}_{44}| &= (\lambda+\mu|\xi|^2)\ \ \{\lambda(\lambda+\mu|\xi|^2) + (\xi_1^2+\xi_2^2)(\lambda\nu+\gamma^2)\}.\end{aligned}$$

Employing the same argument, we can find others minors

$$\begin{aligned}|\widehat{\mathbb{A}}_{23}| &= (\lambda+\mu|\xi|^2)\{(\xi_1\xi_2)(\lambda\nu+\gamma^2)\} = |\widehat{\mathbb{A}}_{32}|, \\ |\widehat{\mathbb{A}}_{24}| &= -(\lambda+\mu|\xi|^2)\{(\xi_1\xi_3)(\lambda\nu+\gamma^2)\} = |\widehat{\mathbb{A}}_{42}|, \\ |\widehat{\mathbb{A}}_{34}| &= -(\lambda+\mu|\xi|^2)\{(\xi_2\xi_3)(\lambda\nu+\gamma^2)\} = |\widehat{\mathbb{A}}_{32}|.\end{aligned}$$

Moreover, we have the cofactors in the following

$$\begin{aligned}\widehat{C}_{11} &= (\lambda+\mu|\xi|^2)^2\{(\lambda+\mu|\xi|^2)+\nu|\xi|^2\}, \\ \widehat{C}_{1j} &= \widehat{C}_{j1} - (i\gamma\xi_{j-1})(\lambda+\mu|\xi|^2)^2, \\ \widehat{C}_{ij} &= \widehat{C}_{ji}(\lambda+\mu|\xi|^2)\{\lambda(\lambda+\mu|\xi|^2)\delta_{ij} + (\delta_{ij} - \xi_{i-1}\xi_{j-1})(\lambda\nu+\gamma^2)\}\end{aligned}$$

with $\delta_{ij} = 0$ for $i \neq j$, $\delta_{ij} = 1$ for $i = j$ and $i, j = 2, 3, 4$.

4. Proof of Theorem

Throughout this section, we use the notation introduced in the previous section.

4.1 Eigen values

In this subsection, we investigate the eigen values. The solution formula for the model problem has already obtained by Kobayashi [5]. Here, we shall give a slightly detail how to get the solution formula for velocity dan the density. First of all, we determine the determinant of matrix $\det[\lambda I + \widehat{\mathbb{M}}(\xi)] = 0$. By equation (7), we have

$$\det[\lambda I + \widehat{\mathbb{M}}(\xi)] = 0$$

$$\left(\boldsymbol{\lambda} + \boldsymbol{\mu}|\boldsymbol{\xi}|^2\right)^2\left\{\boldsymbol{\lambda}^2 + (\boldsymbol{\mu} + \boldsymbol{\nu})|\boldsymbol{\xi}|^2\boldsymbol{\lambda} + \boldsymbol{\gamma}^2|\boldsymbol{\xi}|^2\right\} = \mathbf{0} \tag{11}$$

From equation (9), we have two possibilities sero values, that are $(\lambda + \mu|\xi|^2)^2 = 0$ or $\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} = 0$. For the first case, we have $\lambda_3 = \lambda_4 = -\mu|\xi|^2$. Furthermore, we will find the eigen values of $\{\lambda^2 + (\mu + \nu)|\xi|^2\lambda + \gamma^2|\xi|^2\} = 0$. By using the formula

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

we have

$$\lambda_{1,2} = \frac{-(\mu+\nu)|\xi|^2 \pm |\xi|\sqrt{(\mu+\nu)^2|\xi|^2 - 4\gamma^2}}{2}. \tag{12}$$

In view of equation (10), for $|\xi| \geq \frac{2\gamma}{(\mu+\nu)}$, we have

$$\lambda_1 = \frac{-(\mu + \nu)}{2}|\xi|^2 + \frac{1}{2}|\xi|\sqrt{(\mu + \nu)^2|\xi|^2 - 4\gamma^2}$$
$$\lambda_2 = \frac{-(\mu + \nu)}{2}|\xi|^2 - \frac{1}{2}|\xi|\sqrt{(\mu + \nu)^2|\xi|^2 - 4\gamma^2}.$$

Meanwhile, for $|\xi| \leq \frac{2\gamma}{(\mu+\nu)}$, we have

$$\lambda_1 = \overline{\lambda_2}\ \frac{-(\mu + \nu)}{2}|\xi|^2 + \frac{i}{2}|\xi|\sqrt{4\gamma^2 - (\mu + \nu)^2|\xi|^2}.$$

Moreover, for $|\xi| = \frac{2\gamma}{(\mu+\nu)}$ can be been in Kobayashi [5]. Thus, we may omit the calculation.

4.2 Fourier transform of $\hat{\rho}$ and $\hat{\boldsymbol{v}}$

In this subsection we consider the formula of $\hat{\rho}$ and $\hat{\mathbf{v}}$, density and velocity, respectively. These density and velocity are the result of the model problem (1). First of all, applying div to second equation of (5), we have

$$\frac{\partial}{\partial t}\text{div}\,\mathbf{v} - \mu\Delta\text{div}\,\mathbf{v} - \nu\Delta\text{div}\,\mathbf{v} + \gamma\Delta\rho = 0. \tag{13}$$

Let $D = \text{div v}$, then equation (13) can be written as follows

$$D_t - \omega\Delta D + \gamma\Delta\rho = 0,$$

with $\omega = \mu + \nu$. Recalling first equation of (3),

$$\rho_t = -\gamma\text{div v},$$

then we can write the equation to be

$$\rho_t = -\gamma D,$$
$$\frac{\rho_t}{\gamma} = -D. \tag{14}$$

Here in after, we differentiate the equation of (14) recpect to t variable and substitute (11) and (12) to the result, we obtain

$$\frac{1}{\gamma}\rho_{tt} = -D_t,$$
$$\frac{1}{\gamma}\rho_{tt} = \gamma\,\Delta\,\rho - \omega\Delta D,$$
$$\frac{1}{\gamma}\rho_{tt} = \gamma\,\Delta\,\rho - \omega\left(-\Delta\frac{\rho_t}{\gamma}\right),$$
$$\rho_{tt} = \gamma^2\,\Delta\,\rho + \omega\Delta\rho_t. \tag{15}$$

Applying Fourier transform to equation (15), we have

$$\hat{\rho}_{tt} + \omega|\xi|^2\hat{\rho}_t + \gamma^2\,|\xi|^2\,\hat{\rho} = 0 \tag{16}$$

with the initial condition

$$\hat{\rho}(\xi,0)=\hat{\rho}_0(\xi),\ \ \hat{\rho}_t(\xi,0)=-i\xi\hat{\mathbf{v}}_0(\xi). \tag{17}$$

Moreover, we have the general solution for equation (16)

$$\hat{\rho}(\xi,t)=c_1e^{\lambda_1(\xi)t}+c_2e^{\lambda_2(\xi)t}, \tag{18}$$

where

$$\lambda_{1,2}=\frac{-\omega|\xi|^2\pm|\xi|\sqrt{\omega^2|\xi|^2-4\gamma^2}}{2}.$$

Substituting the initial condition (17) to (18), we obtain

$$c_1=\frac{\lambda_2(\xi)\hat{\rho}_0(\xi)+i\xi\hat{\mathbf{v}}_\mathbf{0}(\xi)}{\lambda_2(\xi)-\lambda_1(\xi)},\qquad c_2=-\frac{\lambda_1(\xi)\hat{\rho}_0(\xi)+i\xi\hat{\mathbf{v}}_0(\xi)}{\lambda_2(\xi)-\lambda_1(\xi)}.$$

Therefore, we have the soltion formula for

$$\hat{\rho}(\xi,t)=\left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)t}-\lambda_1(\xi)e^{\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{\lambda_2(\xi)t}-e^{\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi). \tag{19}$$

Furthermore, we determine te solution formula for $\hat{\mathbf{v}}(\xi,t)$. Employing the same argument in [3, Section 3], firstly applying Fourier transform to the second equation of (5), we obtain

$$\hat{\mathbf{v}}_\mathrm{t}-\mu|\xi|^2\hat{\mathbf{v}}-\mathrm{i}\nu\xi_\mathrm{j}\sum_{k=1}^{3}\mathrm{i}\xi_\mathrm{k}\widehat{v_k}+i\gamma\xi_\mathrm{j}\hat{\rho}=0, \tag{20}$$

We can write equation (20) in the following sense,

$$\hat{\mathbf{v}}_t=(-\mu|\xi|^2\mathbf{I}-\nu\xi\xi^T)\hat{\mathbf{v}}-i\gamma\xi\hat{\rho}. \tag{21}$$

Vector $\hat{\mathbf{v}}(\xi,t)$ is a vector which parallel and orthogonal from ξ, so that we can write the vector $\hat{\mathbf{v}}(\xi,t)$ as follows

$$\hat{\mathbf{v}}(\xi,t)=a(\xi,t)\frac{\xi}{|\xi|}+b(\xi,t), \tag{22}$$

Where $b(\xi,t)$ orthogonal to ξ, and $a(\xi,t)$ is a scalar such that $a(\xi,t)=\hat{\mathrm{v}}(\xi,t)\frac{\xi}{|\xi|}$.

Substituting equation (22) to (21), then we have

$$\begin{aligned}\hat{\mathbf{v}}_t(\xi,t)&=(-\mu|\xi|^2\mathrm{I}-\nu\xi\xi^T)\left(a(\xi,t)\frac{\xi}{|\xi|}+b(\xi,t)\right)-i\gamma\xi\hat{\rho},\\&=-a(\xi,t)\frac{\xi}{|\xi|}(\mu|\xi|^2\mathrm{I}+\nu\xi\xi^T)-b(\xi,t)(\mu|\xi|^2\mathrm{I}+\nu\xi\xi^T)-i\gamma\xi\hat{\rho},\\&=-a(\xi,t)\left(\mu|\xi|\xi+\nu\frac{\xi\cdot\xi\xi^T}{|\xi|}\right)-b(\xi,t)(\mu|\xi|^2+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}.\end{aligned} \tag{23}$$

Next, we differentiate equation (20) and then substitute the result to the left hand-side of (21), we have

$$a_t(\xi,t)\frac{\xi}{|\xi|}+b_t(\xi,t)=-a(\xi,t)\left(\mu|\xi|\xi+\nu\frac{\xi\cdot\xi\xi^T}{|\xi|}\right)-b(\xi,t)(\mu|\xi|^2+\nu\xi\xi^T)-i\gamma\xi\hat{\rho}.$$

Therefore, we obtain

$$a_t(\xi,t)=\omega|\xi|^2a(\xi,t)-i\gamma|\xi|\hat{\rho},\quad b_t(\xi,t)=-\mu|\xi|^2\,b(\xi,t). \tag{24}$$

By (23) and the initial condition $b(\xi,0)=\left(\mathrm{I}-\frac{\xi\xi^T}{|\xi|^2}\right)\hat{\mathbf{v}}_0(\xi)$, we see that

$$b(\xi,t)=e^{\mu|\xi|^2t}\left(\mathrm{I}-\frac{\xi\xi^T}{|\xi|^2}\right)\hat{\mathbf{v}}_0(\xi). \tag{25}$$

Also, by applying integrating factor to first equation (24), we obtain

$$a(\xi,t)=e^{-\omega|\xi|^2t}\left(a(\xi,0)-i\,\gamma|\xi|\int_0^t e^{\omega|\xi|^2s}\,\hat{\rho}(\xi,s)ds\right)$$

with $a(\xi,0)$ constant.

Furthermore, we will investigate the formula of the second term equation (26). Multiplying (19) by $e^{\omega|\xi|^2s}$, we have

$$\begin{aligned} e^{\omega|\xi|^2s}\,\hat{\rho}(\xi,s) &= \left(\frac{\lambda_2(\xi)e^{\lambda_1(\xi)s+\omega|\xi|^2s}-\lambda_1(\xi)e^{\lambda_2(\xi)s+\omega|\xi|^2s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi) \\ &\quad - i\left(\frac{e^{\lambda_2(\xi)s+\omega|\xi|^2s}-e^{\lambda_1(\xi)s+\omega|\xi|^2s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \\ &= \left(\frac{\lambda_2(\xi)e^{-\lambda_2(\xi)s}-\lambda_1(\xi)e^{-\lambda_1(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)-i\left(\frac{e^{-\lambda_1(\xi)s}-e^{-\lambda_2(\xi)s}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \end{aligned} \tag{26}$$

since $\lambda_{1,2}(\xi)+\omega|\xi|^2=-\lambda_{2,1}(\xi)$ and $\lambda_1(\xi)\lambda_2(\xi)=\gamma|\xi|^2$.
By integrating (26) from $0\le s\le t$, we have

$$\begin{aligned} &\int_0^t e^{\omega|\xi|^2s}\,\hat{\rho}(\xi,s)\,ds \\ &\qquad = \left(\frac{-e^{-\lambda_2(\xi)t}-e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi) \\ &\qquad + \left(\frac{i}{\gamma|\xi|^2}\right)\left(\frac{\lambda_2(\xi)e^{-\lambda_1(\xi)t}-\lambda_1(\xi)e^{-\lambda_2(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\xi\hat{\mathbf{v}}_0(\xi), \end{aligned}$$

without loss of generality, we take $a(\xi,0)=0$ so that $e^{\omega|\xi|^2t}\,a(\xi,0)=0$. Thus, we have

$$a(\xi,t)=-i\gamma|\xi|\left(\frac{-e^{-\lambda_2(\xi)t}-e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\hat{\rho}_0(\xi)+\left(\frac{\lambda_2(\xi)e^{-\lambda_2(\xi)t}-\lambda_1(\xi)e^{-\lambda_1(\xi)t}}{\lambda_2(\xi)-\lambda_1(\xi)}\right)\frac{\xi\hat{\mathbf{v}}_0(\xi)}{|\xi|}. \tag{27}$$

Substituting (24) and (27) to (20), this complete the proof of the **Theorem 1.2**.

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