

The Graphically Analysis of the Power of Hypothesis Parameters on Discrete (Binomial) and Continuous (Gamma) Distributions

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Abstract

The research studied power of hypothesis parameters on Binomial (discrete) and Gamma (continuous) distributions. Here, we derived the rejection area using uniformly most powerful test (UMPT) to find the formula of the Power. The *R-code* is then used to figure and graphically analysis the graph. The result showed that the curves are depended on the parameter shape, sample size (small n), and K_0 .

Keywords:

The Curve, power, parameter shape, and R-code

1. Introduction

In the hypothesis testing, we have a probability error type I (α), a probability error type II (β) and a **power**. Later, power has significantly contribute to draw the improving population parameter. The power is then defined as a probability to reject H_0 under H_1 in testing hypothesis, $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$, for parameter θ (Wackerly, et al., 2008). We then noted that one of the statistical techniques to investigate the population inference, namely power and size. Here, we noted that the power and size now begin to be used to conclude the population inference, especially in parameter testing. Following Wackerly, et al. (2008) and Casella and Berger (2002), we the got the more detail of the definition of the power and size, that are, power is the probability to reject H_0 under H_1 and the size is the probability to reject H_0 under H_0 .

Many authors already studied the improving inference population using power and size such as Pratikno et al. (2017, 2019, 2020), Pratikno (2012), Khan and Pratikno (2013), Khan (2003), Khan and Saleh (1995), Khan and Hoque (2003), Saleh (2006), Yunus (2010), and Yunus and Khan (2007). Here, Pratikno [2012], Khan and Pratikno [2013] and Khan [2003] studied about a testing intercept using non-sample prior information (NSPI). Moreover, Pratikno [2012] and Khan et al. [2016] used the power and size to compute the cdf of the bivariate noncentral F (BNCF) distribution of the pre-test test (PTT) with NSPI in some regression models. The computational of the probability integral of the probability distribution function (pdf) and cdf of the BNCF distribution are very complicated and hard, so the *R* code is used.

In the context of the power on the distribution, Pratikno et al. (2017, 2019, 2020), are already presented the formula and value of the power on discrete and continuous distributions. Here, we then focused to analysis (research) about the power and size on Binomial (discrete) and Gamma (continuous) distribution. There are several steps to compute the power of the distribution: (1) determine the sufficiently statistics, (2) compute the rejection

area using *uniformly most powerful test* (UMPT), (3) derive the formula of the power of the distributions in testing hypothesis, and (4) plot the graphs of the power of their distributions.

In this paper, Section 1 presented the introduction. The methodology research is given in Section 2. A simulation data (graph of the power and size) is obtained in Section 3, and Section 4 described the conclusion of the research.

2. Research Methodology

Step 1. We studied Binomial and Gamma Distribution.

Step 2. We then refer to previous research to derived the formula of the power and size .

Step 3. Following step 2, we then produced and graphically analysis of the graph of the power

Step 4. Finally, we draw the conclusion by choosing the maximum power and minimum size for both distributions

3. Discussion Results

3.1 The Power on the Binomial Distribution

In this section, we follow Pratikno et al. (2020) in discussing the power of the Binomial distribution. Here, we

let X_i Bernoulli case with parameter θ , $n = 20$, and $Y = \sum_{j=1}^{n=20} X_j$, then Y is written as $Y : \text{Bin}(n, \theta)$ follows

Binomial distribution with $n = 20$ and θ , as independent random variable with *probability mass function* (PMF) is given as

$$f(x_i) = \binom{n}{x} \theta^x (1 - \theta)^{n-x} . \quad (1)$$

For testing the two-side hypothesis, $H_0 : \theta = 0.1$ versus $H_1 : \theta \neq 0.1$ on rejection area $\{(x_1, \dots, x_{12}) : Y \leq 6\}$, we then derive the formula of the power as given below

$$\begin{aligned} \pi(\theta) &= P(\text{Reject } H_0 | \text{under } H_1) = \sum_{y=0}^6 \binom{20}{y} \theta^y (1 - \theta)^{20-y} \\ &= \binom{20}{0} \theta^0 (1 - \theta)^{20-0} + \binom{20}{1} \theta^1 (1 - \theta)^{20-1} + \dots + \binom{20}{6} \theta^6 (1 - \theta)^{20-6} \\ &= \left(\frac{20!}{0!(20-0)!} \right) 1(1 - \theta)^{20} + \left(\frac{20!}{1!(20-1)!} \right) \theta^1 (1 - \theta)^{19} + \dots + \left(\frac{20!}{6!(20-6)!} \right) \theta^6 (1 - \theta)^{14} \\ &= (1 - \theta)^{20} + (20) \theta^1 (1 - \theta)^{19} + \dots + (38760) \theta^6 (1 - \theta)^{14} . \end{aligned} \quad (2)$$

The size in testing the two-side hypothesis, $H_0 : \theta = 0.1$ versus $H_1 : \theta \neq 0.1$ on rejection area $\{(x_1, \dots, x_{12}) : Y \leq 6\}$, is then given as

$$\alpha(\theta) = P(\text{Reject } H_0 | \text{under } H_0) = \sum_{y=0}^6 \binom{20}{y} \theta^y (1-\theta)^{20-y} \Big|_{\theta=0.1}$$

$$= (1-0.1)^{20} + (20)(0.1)^1(1-0.1)^{19} + \dots + (38760)(0.1)^6(1-0.1)^{14} \quad (3)$$

From the equation (3), it is clear that the size is constant, so the graph is not depended on parameter shape, but it will be straight line.

Using the equation (2) and following Pratikno et al. (2020), we then produce the graph of power of the Binomial distribution. The graph is given in Figure 1.

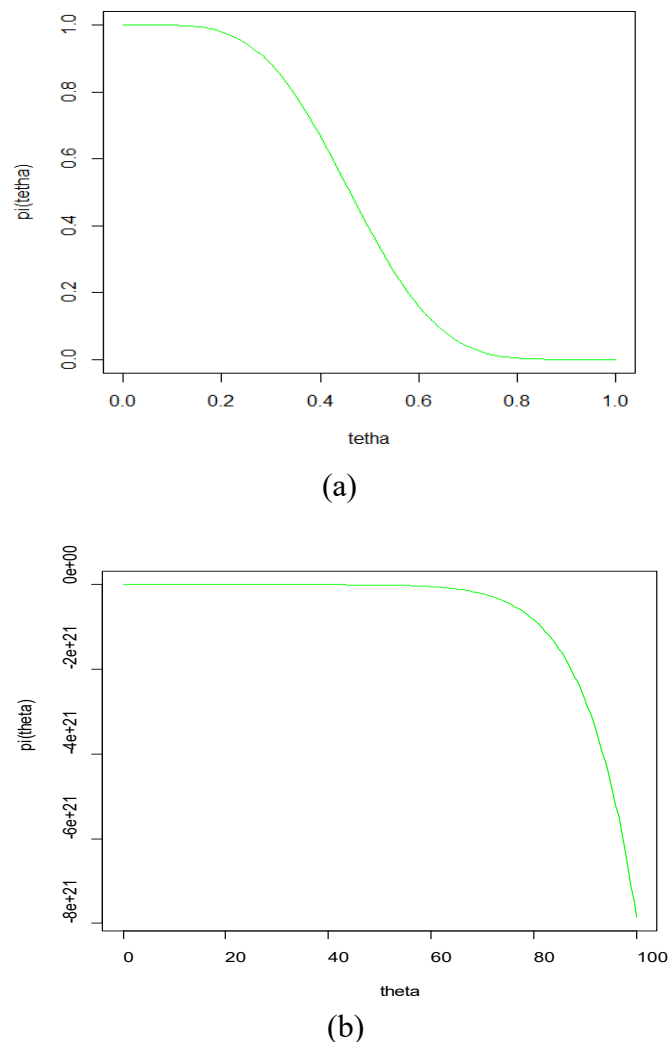


Figure 1. The Power of the Binomial Distribution

From Figure 1., we see that the curves are similar (sigmoid and $\frac{1}{2}$ sigmoid), the curve on the graph (a) tend to be one in short period on the parameter, that is $0 \leq \theta \leq 0.2$, and they begin decrease on $0.2 \leq \theta \leq 0.8$ (sigmoid), and then they tend to be flat (asymptote to zero) on $\theta \geq 0.8$. Here, we note that the period of the probability one of the curve on the graph (b) is more length, that is $0 \leq \theta \leq 0.8$. This is due to the different value of the null hypothesis. Generally, we note that they (both of the graphs) will be maximum on one, and the minimum is zero.

Following the previous research of Pratikno (2012), Khan and Pratikno (2013), Khan (2003), Khan and Saleh (1995), Khan and Hoque (2003), Saleh (2006), Yunus (2010), and Yunus and Khan (2007), we noted that the maximum power (one) and minimum size are chosen for the better indicator testing. I therefore conclude that the graph of the power will be not so similar (or the same as) when n , θ , and rejection area are different. When they increase (or decrease), the graph will be different in achieving of the maximum power going to be one.

3.2 The Power on the Gamma Distribution

Following the previous research of the Pratikno (2012) and Pratikno et al. (2019, 2020) and the probability distribution function (pdf) of Gamma distribution of the variable random X with parameter $\kappa > 0$ and $\theta > 0$

$f(x, \kappa, \theta) = \frac{1}{\theta^\kappa \Gamma(\kappa)} x^{\kappa-1} e^{-x/\theta}$, $x > 0$, the formula of the power of the Gamma distribution in testing

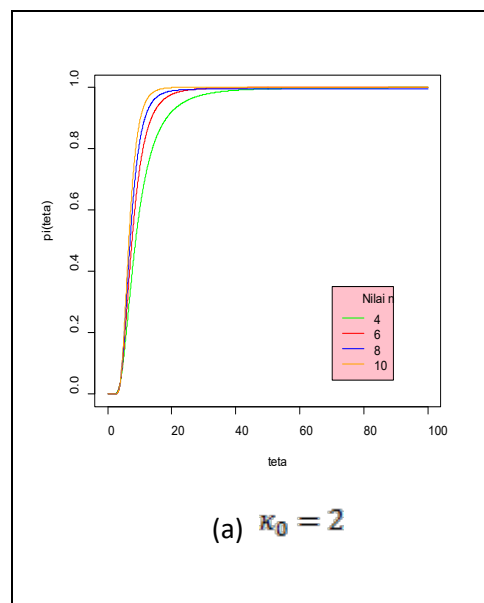
$H_0 : \theta = 0.1$ versus $H_1 : \theta > 0.1$, is then given as

$$\pi(\theta) = 1 - P\left(\frac{2}{\theta} \sum_{i=1}^n x_i \leq \frac{\theta_0}{\theta} \chi^2_{(2n\kappa_0; \alpha)}\right) = 1 - \int_0^{\frac{\theta_0}{\theta} \chi^2_{(2n\kappa_0; \alpha)}} \frac{1}{2^n \Gamma(n)} x^{n-1} e^{-\frac{x}{2}} dx. \quad (4)$$

Similarly, the formula of the size of the Gamma distribution is then written, for $\theta = 0.1$, as

$$\alpha(\theta) = 1 - P\left(\frac{2}{\theta} \sum_{i=1}^n x_i \leq \frac{\theta_0}{\theta} \chi^2_{(2n\kappa_0; \alpha)}\right) = 1 - \int_0^{\frac{\theta_0}{\theta} \chi^2_{(2n\kappa_0; \alpha)}} \frac{1}{2^n \Gamma(n)} x^{n-1} e^{-\frac{x}{2}} dx \quad (5)$$

Here, the value of the θ on the equation (5), is exactly 0.1. So the curve is not sigmoid, but it will be straight line and constant. Using the equation (4), we then simulate on several n and κ_0 for $\theta_0 = 2$ and $\alpha = 0,01$ to get the graphs of the power. The graph of the power of the Gamma distribution is then given in Figure 2.



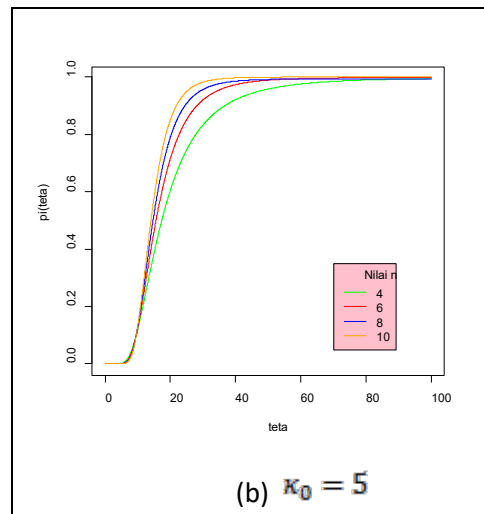


Figure 2. The Power of the Gamma Distribution

From Figure 2 (a) and (b), we see that all the curves are influenced by n and K_0 . The highest curve is occurred on large n and small K_0 (see both Figure 2). We also noted that the curves will be move to the right (more skew to the right) for large K_0 , and the lower curve is occurred when they have small n . Generally, all the curve tend to be one more faster when the parameter shape is small, and they will go to be one on short period of the parameter shape. Here, we note that the 1st graph have short period of the parameter shape between 0 to 0.2, but the 2nd graph is around 0 to 0.4. It is clear that they tend to be smooth (skew to the right) when n and K_0 are increase. We also noted that they are more faster to one for small n and K_0 . Here, we also note that both of the graphs (a) and (b) will be maximum on one, and the minimum is zero in small parameter shape. Following the previous research of such as Pratikno (2012), Khan and Pratikno (2013), Khan (2003), Khan and Saleh (1995), we must to chose the maximum power (one) as an indicator testing.

4 Conclusion

The research studied power and size on Binomial (discrete) and Gamma (continuous) distribution. The rejection area is used to find the formula of the power, and the *R-code* is then used to figure and graphically analysis. The result showed that the influencing factor of the curves of the power are parameters shape and sample size (n), and we also note that all the curve tend to be more faster to one for small n and K_0 .

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Biographies

Budi Pratikno, currently works as a lecturer in Department of Mathematics, Universitas Jenderal Soedirman, author of several international journals related to testing intercepts with non-sample prior information (NSPI), which are widely published in international journals such as Statistical Papers, Far East Journal of Mathematics Science, Springer, JSTA, ISSOS, IJET, IJAST, IOP series, IEOM and others. In the study of NSPI as the main research the author, we try to improve population inference using non-sample. The author completed his undergraduate education at the Mathematics Department (UGM), then continued his Masters in Statistical Science at La Trobe University, Melbourne, Australia, and his S3 in Statistical Science (Ph.D) at the University of Southern Queensland (USQ), Toowoomba, Brisbane, Australia.

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