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Solution of the Second Order of the Linear Hyperbolic Equation Using Cubic B-Spline Collocation Numerical Method

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ABSTRACT

Wave equation is one of the second order of the linear hyperbolic equation. Telegraph equation as a special case of wave equation has interesting point to investigate in the numerical point of view. In this paper, we consider the numerical methods for one dimensional telegraph equation by using cubic B-spline collocation method. Collocation method is one method to solve the partial differential equation model problem. Cubic spline interpolation is an interpolation to a third order polynomial. This polynomial interpolate four point. B-Spline is one of spline function which related to smoothness of the partition. For every spline function with given order can be written as linear combination of those B-spline. As we known that the result of the numerical technique has difference with the exact result which we called as, so that we have an error. The numerical results are compared with the interpolating scaling function method which investigated by Lakestani and Saray in 2010. This numerical methods compared to exact solution by using RMSE (*root mean square error*), L_2 norm error and L_∞ norm error. The error of the solution showed that with the certain function, the cubic collocation of numerical method can be used as an alternative methods to find the solution of the linear hyperbolic of the PDE. The advantages of this study, we can choose the best model of the numerical method for solving the hyperbolic type of PDE. This cubic B-spline collocation method is more efficiently if the error is relatively small and closes to zero. This accuration verified by test of example 1 and example 2 which applied to the model problem.

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A. INTRODUCTION

Wave propagation in cable transmission can be described in mathematical modelling. This model can be written in partial differential equations. One example is wave equation not only in one dimension but also two dimensional case (Dosti & Nazemi, 2012). We know that telegraph is one of the wave equations. In this article we consider the solution formula of the telegraph equation as a second order of the linear hyperbolic equation problem. A model for second order of one dimensional linear hyperbolic equation is described in the following:

$$u_{tt} + 2\alpha u_t + \beta^2 u = u_{xx} + f(x,t),\ a \le x \le b,\ t \ge 0 \quad (1)$$

with initial condition and boundary condition are

$$\begin{cases} u(x,0) = f_0(x), & a \leq x \leq b \\ u_t(x,0) = f_1(x), & a \leq x \leq b \end{cases} \tag{2}$$

and

$$\begin{cases} u(a,t) = g_0(t), & t \geq 0 \\ u_t(b,t) = g_1(t), & t \geq 0 \end{cases} \tag{3}$$

Respectively. Here α, β are positive constant coefficients, $f_0(x), f_1(x)$ and their derivatives are continuous function with respect to x variable, and also $g_0(t), g_1(t)$ and their derivatives are continuous function with respect t variable. A derivative model can be solved in mathematical analytic or numerical analytic point of view. However, nota ll derivative model can be solved in analytic poin of view. We known that the exact solution is the real solution and has no errors. In general, the solution of the telegraph equation is investgate b by using numerical point of view because of the non-homogenous part.

As we known that the partial differential equation of the hyperbolic model becomes basis of atomic physics which is the fundamental equation and also the vibrations of the structures. The examples of this PDE models are buildings, beams and machines. Equation (1) known as second-order telegraph equation with constant coefficient. This formula can be seen in (Sharifi & Rashidinia, 2016). Recently, there are many researcher investigated telegraph equation not only using the numerical methods but also by using mathematical analysis approach. Lakestani & Saray (2010) studied the numerical solution of the telegraph equation using interpolating scaling function. Meanwhile, Dosti & Nazemi (2012) solved telegraph equation using B-spline quasi interpolation methods.

In the derivation of cubic B-spline collocation method, the examples were used in (Lakestani & Saray, 2010) are different with the article. In this article, the numerical results are compared with another numerical methods which studied by Sharifi & Rashidinia (2016). (Mittal & Jain, 2012) investigated similar numerical methods which is applied to convection-diffusion equation. They studied for the Neumann's boundary conditions.

Many researcher studied the solution of the telegraph equation in mathematical analysis point of view. Chen et al (2008) investigated the time-fractional telegraph equation by using separating variable method. Meanwhile, Das et al (2011) investigated the time fractional of telegraph equation in mathematical analytic point of view. On the other hand, Wang et al., (2020) studied the solution of the telegraph equations by using fractal derivative. Biazar & Eslami (2010) considered the solution of the telegraph equation by using differential transform method (DTM). This method can find the exact solution or a closed approximate solution of an equation. Atangana (2015) investigated not only the stability but also the convergence of the time -fractional variable order of the telegraph equation.

In contrast, the solution of the telegraph equation in the numerical point of view have been studied by many authors. Jiwari et al (2012) investigated the numerical method based on differential quadrature method (PDQM) for hyperbolic partial differential equation type of the vibration structures such as buildings, beams and machines. Two years before, Saadatmandi & Dehghan (2010) studied the numerical scheme to solve the one-dimensional hyperbolic telegraph equation by expanding the approximation of the solution as the elements of shifted Chebyshev polynomial. Hosseini et al (2014) focused on the coupled of the radial basis functions and finite difference scheme achieve the semi-discrete solution. The

Laguerre wavelet collocation method for the fractional-order dimensional telegraph equation (Srinivasa & Rezazadeh, 2021). Furthermore, to see the numerical method which based on the boundary integral equation (BIE) and also the application of the dual reciprocity method (DRM) is explained in (Dehghan & Ghesmati, 2010), while Dehghan & Salehi (2012) investigated the RBF solution of second-order two-space dimensional linear hyperbolic telegraph equation. The extended cubic B-spline method for the solution of time fractional telegraph is discussed in (Akram et al., 2019). Rashidinia & Jokar (2016) presented the polynomial scaling functions to solve the second-order one space-dimensional hyperbolic telegraph equation, and wave propagation of electric signals in a cable transmission line by using homotopy perturbation method (HPM) (Javidi & Nyamoradi, 2013). The purpose of this research, we can see that the cubic collocation numerical method are better used as an alternative methods for the hyperbolic partial differential equations. Before we state our main result in the next session, we introduce our notation used throughout the paper.

Notation $\mathbb{N}$ denotes the sets of natural numbers and we set $\mathbb{N}_{\mathbf{0}} = \mathbb{N} \cup \{\mathbf{0}\}$**.** $\mathbb{C}$ and $\mathbb{R}$ denote the sets of complex numbers and real numbers, respectively. For any multi-index $\kappa = (\kappa_1, \dots, \kappa_N) \in \mathbb{N}_{\mathbf{0}}^{\boldsymbol{N}}$, we write $|\kappa| = \kappa_1 + \cdots + \kappa_N$ and $\partial_x^{\kappa} = \partial_1^{\kappa_1} \cdots \partial_N^{\kappa_N}$ with $\mathrm{x} = (\mathrm{x}_1, \dots, \mathrm{x}_\mathrm{N})$. For $\mathrm{N} \times \mathrm{N}$ matrices of function $\mathbf{F} = (F_{ij})$. We use capital boldface letters, e.g. **A** to denote matrix-valued functions. But, we also use the Greek letters, e.g α, β, γ such as positive constants.

B. METHODS

The research methodology which used in this paper is literature review of the related articles. In this article, we define the solution of the telegraph equation in numerical methods point of view. The procedures are in the following, first of all, we discretise equation of the telegraph equations not only the model problem but also the initial condition and boundary conditions by using finite difference approximation. The second step, we apply the cubic B-spline collocation methods to the model problem and its initial and boundary conditions. Furthermore, the simulation of the solution are applied for each criteria. Then, we measure the error of the solution by L_∞-error, L_2-error and root mean square error (RMSE). The technical of the B-spline collocation methods follow in (Sharifi & Rashidinia, 2016). Meanwhile, the technique applying the numerical method of B-spline collocation methods are followed Dosti & Nazemi (2012). For the simulation we use matlab 7.0.4 software. In the following section, we explained more detailed.

C. RESULT AND DISCUSSION

1. Description of the Numerical Method

In this subsection, we consider a finite difference approximation to discretise equation (1). A we known that the equation system of partial differential equations (PDE) can be solved with numerical methods. There are many numerical methods to solve the PDE. In this paper, we are focusing on the cubic B-spline collocation methods. However, before we apply that numerical methods, first of all we consider the discretisation the model problem. Since the numerical methods are a method to approximate the solution of the PDE, then we need a simulation algorithm to make sure that the solution which we get are more effective. In the following are the discretisation of t and x variables in time and space, respectively.

$$(u_{tt})_i^j = \frac{u_i^{j+1} - 2u_i^j + u_i^{j-1}}{k^2}$$

$$(u_t)_i^j = \frac{u_i^{j+1} - u_i^{j-1}}{2k}$$

$$(u_{xx})_i^j = \frac{(u_{xx})_i^{j+1} + (u_{xx})_i^{j-1}}{2}. \quad (4)$$

Substitution equation (4) to equation (1) we have

$$\frac{u_i^{j+1} - 2u_i^j + u_i^{j-1}}{k^2} + 2\alpha\frac{u_i^{j+1} - u_i^{j-1}}{2k} + \beta^2 u_i^j = \frac{(u_{xx})_i^{j+1} + (u_{xx})_i^{j-1}}{2} + f(x_i, t_j). \quad (5)$$

By simplicity, we can write the equation (5) to be

$$(1 + \alpha k)u_i^{j+1} - \frac{k^2}{2}(u_{xx})_i^{j+1} = r_i(x), \quad (6)$$

where

$$r_i(x) = \frac{k^2}{2}(u_{xx})_i^{j-1} + k^2 f(x_i, t_j) - (\beta^2 k^2 - 2)u_i^j - (1 - \alpha k)u_i^{j-1}. \quad (7)$$

Furthermore, by using Taylor series expansion, we can calculate u_i^1 with the formula

$$u_i^1 = u_i^0 + k(u_t)_i^0 + \frac{k}{2!}(u_{tt})_i^0 + R_3(x), \quad (8)$$

where u_i^0 and $(u_t)_i^0$ are the initial conditions (2). We also get formula of $(u_{tt})_i^0$ in the following:

$$(u_{tt})_i^0 = (u_{xx})_i^0 + f(x_i, t_j) - 2\alpha(u_t)_i^0 - \beta^2 u_i^0. \quad (9)$$

Meanwhile, we defined $(u_{xx})_i^j$ using another finite difference approximation in thw following:

$$(u_{xx})_i^j = \frac{u_{i+1}^j - 2u_i^j + u_{i-1}^j}{h^2}. \quad (10)$$

Moreover, substituting (10) to (7), then (9) to (8), we have new formula of u_i^1 i.e

$$u_i^1 = f_0(x_i) + kf_1(x_i) + \frac{k}{2!}\left(\frac{u_{i+1}^0 - 2u_i^0 + u_{i-1}^0}{h^2}\right) + f(x_i, t_j) - 2\alpha(u_t)_i^0 - \beta^2 u_i^0 + R_3(x). \quad (11)$$

Therefore, we have formula for u_i^{j+1}

$$u_i^{j+1} = \left(\frac{2h^2 - k^2h^2\beta^2 - 2k^2}{h^2 + \alpha kh^2}\right)u_i^j + \left(\frac{\alpha kh^2 - h^2}{h^2 + \alpha kh^2}\right)u_i^{j-1} + \frac{k^2}{h^2 + \alpha kh^2}(u_{i+1}^j + u_{i-1}^j)$$

$$+\frac{k^2\,h^2}{h^2+\alpha k h^2}f(x_i,t_j). \tag{12}$$

2. Cubic B-Spline Collocation Solution

a. Approximation solution of boundary value problem

We define cubic B-spline in the following (PM, 1975)

$$B_{4,i}(x)=\frac{1}{6h^3}\begin{cases}(x-x_{i-2})^3 & x\in[x_{i-2,}x_{i-1})\\ h^3+3h^2(x-x_{i-1})+3h(x-x_{i-1})^2-3(x-x_{i-1})^3 & x\in[x_{i-1,}x_i)\\ h^3+3h^2(x_{i+1}-x)+3h(x_{i+1}-x)^2-3(x_{i+1}-x)^3 & x\in[x_{i,}x_{i+1})\\ (x_{i+1}-x)^3 & x\in[x_{i+1,}x_{i+2})\\ 0 & x \text{ other}\end{cases} \tag{13}$$

$B_{4,i}(x)$ value and its derivatives at the nodal points can be seen in the following as shown in Table 1.

Table 1. $B_{4,i}(x)$ value and its derivatives at the nodal points

x	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}
$B_{4,i}(x)$	0	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$	0
$B_{4,i}{}'(x)$	0	$\frac{1}{2h}$	0	$-\frac{1}{2h}$	0
$B_{4,i}{}''(x)$	0	$\frac{1}{h^2}$	$-\frac{2}{h^2}$	$\frac{1}{h^2}$	0

Developing the numerical method to approximate the solution of the boundary value problem of equation (1) – (3), we define $\hat{S}(x)$ as in (Phillips, 2003)

$$\hat{S}(x)=\sum_{i=-1}^{n+1}c_iB_{4,i}(x), \tag{14}$$

With $c_i(t)$ is a parameter depend on t and it will be calculated by using boundary conditions. Furthermore, we set

$$\mathcal{L}\hat{S}(x_i)=r(x_i),\quad 0\le i\le n \tag{15}$$

$$\hat{S}(x_0)=g_0(t_n),\quad \hat{S}(x_n)=g_1(t_n), \tag{16}$$

where $\mathcal{L}u_i^{j+1}=(1+\alpha k)u_i^{j+1}-\frac{k^2}{2}(u_{xx})_i^{j+1}$.

Moreover, by equation (13) and (14), we have

$$\hat{S}(x)=\frac{1}{6}c_{i-1}+\frac{4}{6}c_i+\frac{1}{6}c_{i+1} \tag{17}$$

$$\hat{S}'(x) = \frac{1}{2h}c_{i-1} - \frac{1}{2h}c_{i+1} \tag{18}$$

$$\hat{S}''(x) = \frac{1}{h^2}c_{i-1} - \frac{2}{h^2}c_i + \frac{1}{h^2}c_{i+1}. \tag{19}$$

Substituting equation (17) and (19) to operator $\mathcal{L}$ we have

$$\left(\frac{1}{6}(1+\alpha k)h^2 - \frac{k^2}{2}\right)c_{i-1} + \left(\frac{4}{6}(1+\alpha k)h^2 + k^2\right)c_i + \left(\frac{1}{6}(1+\alpha k)h^2 - \frac{k^2}{2}\right)c_{i+1} = h^2 r_i(x) \tag{20}$$

where $i = 0, 1, \dots, n$ with c_0 and c_n can be calculated from boundary conditions. Furthermore, we can write equation (20) in the matrix form

$$\mathbf{Ax} = \mathbf{B}$$

with

$$\mathbf{A} = \begin{bmatrix} 3k^2 & 0 & 0 & 0 & \cdots & 0 \\ \gamma - \frac{k^2}{2} & 4\gamma + k^2 & \gamma - \frac{k^2}{2} & 0 & \cdots & 0 \\ 0 & \gamma - \frac{k^2}{2} & 4\gamma + k^2 & \gamma - \frac{k^2}{2} & \cdots & 0 \\ 0 & 0 & \gamma - \frac{k^2}{2} & 4\gamma + k^2 & \gamma - \frac{k^2}{2} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 3k^2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} h^2 r_0(x) + \left(3k^2 - h^2(1+\alpha k)\right)g_0(t_n) \\ h^2 r_1(x) \\ h^2 r_2(x) \\ \vdots \\ h^2 r_{n-1}(x) \\ h^2 r_n(x) + \left(3k^2 - h^2(1+\alpha k)\right)g_n(t_n) \end{bmatrix}$$

where $\gamma = \frac{1}{6}(1+\alpha k)h^2$.

b. Numerical examples

Example 1. We consider equation (1) with the following conditions:

$$f_0(x) = \sinh(x), \quad f_1(x) = -2\sinh(x)$$

$$g_0(t) = 0, \quad g_1(t) = e^{-2t}\sinh(1)$$

and

$$f(x,t) = (3 - 4\alpha + \beta^2)e^{-2t}\sinh(x)$$

and the exact solution is given by

$$u(x,t) = e^{-2t}\sinh(x).$$

We consider the telegraph equation with $\alpha = 4$ and $\beta = 2$ in the interval $0 \le x \le 1$. Initial boundary condition

$$u(\mathrm{x},0)=\sinh(x),\qquad u_t(x,0)=-2\sinh(x)$$

and the boundary conditions

$$u(0,t)=0,\qquad u(1,t)=e^{-2t}\sinh(1).$$

The step sizes of $k=0.001$ and $h=0.002$ for various time $t=0.1,0.2,\ldots,0.5$. The following is Telegraph solution for example 1, as shown in Figure 1.

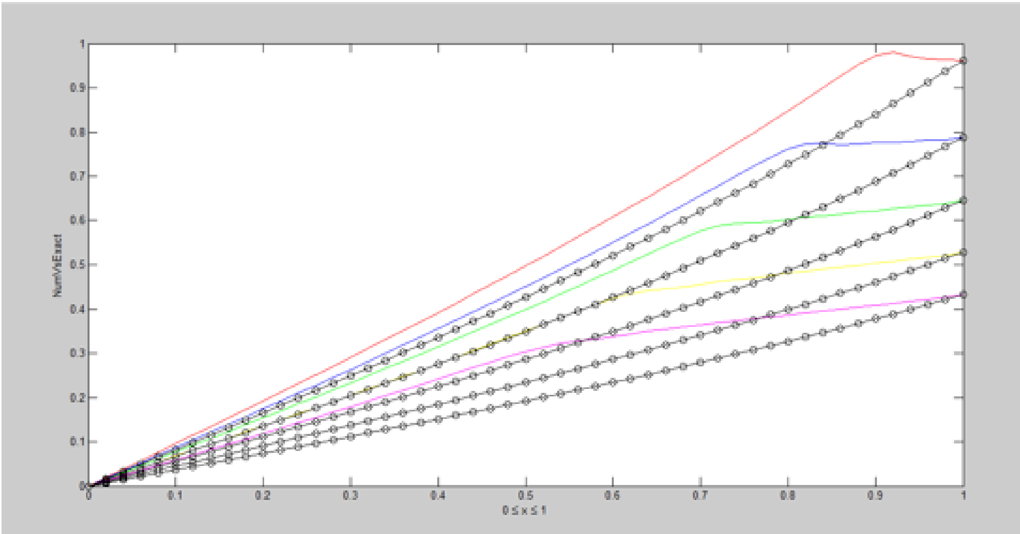

Figure 2. Telegraph solution for example 1

From Figure 1, the black line with circles are the graph of the exact solution. Meanwhile, the blue, yellow and magenta lines are the numerical solution for $t=0.2,0.3,0.4$ and 0.5, respectively. The following is Error values of example 1, as shown in Table 2.

Table 2. Error values of example 1

Time	***RMSE***	***L_2-norm error***	***L_∞-norm error***
$t=0.1$	0.07619	0.07694	0.0134
$t=0.2$	0.09582	0.09677	0.16634
$t=0.3$	0.09312	0.09405	0.15965
$t=0.4$	0.08262	0.08344	0.13832
$t=0.5$	0.07026	0.07096	0.11395

Example 2. We consider equation (1) with the following conditions:

$$f_0(x)=\sin(x),\qquad f_1(x)=0$$

$$g_0(t)=0,\quad g_1(t)=\cos(t)\sinh(1)$$

and

$$f(x,t)=-2\alpha\sin(t)\sin(x)+\beta^2\cos(t)\sin(x)$$

and the exact solution is given by

$$u(x,t) = \cos(t)\sin(x).$$

We consider the telegraph equation with $\alpha = 4$ and $\beta = 2$ in the interval $0 \le x \le 1$. Initial boundary condition

$$u(x,0) = \sin(x), \quad u_t(x,0) = 0$$

and the boundary conditions

$$u(0,t) = 0, \quad u(1,t) = \cos(t)\sin(x).$$

The step sizes of $k = 0.001$ and $h = 0.02$ for various time $t = 0.1, 0.2, \ldots, 0.5$. The following is Telegraph solution for example 2, as shown in Figure 2.

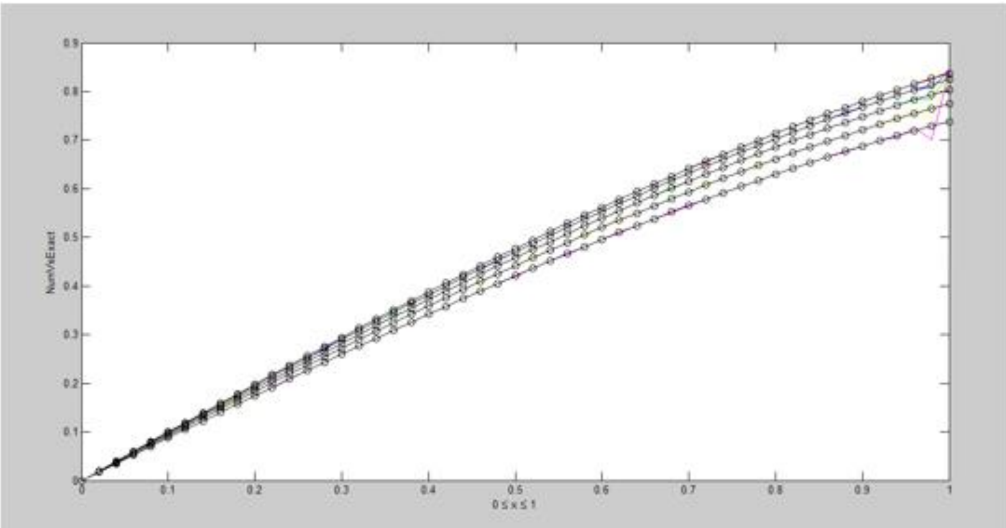

Figure 4. Telegraph solution for example 2

From Figure 1, the black line with circles are the graph of the exact solution. Meanwhile, the blue, yellow and magenta lines are the numerical solution for $t = 0.2, 0.3, 0.4$ and 0.5, respectively. The following is Error values of example 2, as shown in Table 3.

Table 3. Error values of example 2

Time	*RMSE*	*L_2-norm error*	*L_∞-norm error*
$t = 0.1$	0.000074	0.000074	0.0134
$t = 0.2$	0.000119	0.000121	0.16634
$t = 0.3$	0.000166	0.000167	0.15965
$t = 0.4$	0.000213	0.000215	0.13832
$t = 0.5$	0.000256	0.000260	0.11395

From Tables 2 and 3, we can see that the error values of the example 1 and the example 2 are significant for $\alpha = 4$ and $\beta = 2$. Figure 1 and figure 2 are described the comparing result between the exact solution and the numerical solution of the telegraph equation as a simulation. Moreover, the same example in (Lakestani & Saray, 2010) with interpolating scaling method of the numerical approach can solve the problem as effectively as cubic B-spline collocation methods for $0 \leq x \leq 1$, and for variant of $t = 0.2, 0.3, 0.4$ and 0.5.

D. CONCLUSION AND SUGGESTIONS

In this article, we studied the cubic B-spline collocation methods which applied to telegraph equations then we compare the result of Lakestani & Saray (2010). As mention above that in this paper, we use cubic B-spline collocation method while Lakestani and Saray used the interpolating scaling functions. The simulation and illustration for the algorithm are used matlab 7.0.4 series. According to the error values of the solution which are measured by *RMSE*, L_{∞}-norm error and L_2-norm error, its showed that both of the methods are effectively as well.

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REFERENCES

Akram, T., Abbas, M., Ismail, A. I., Ali, N. H. M., & Baleanu, D. (2019). Extended cubic B-splines in the numerical solution of time fractional telegraph equation. *Advances in Difference Equations*, *2019*(1), 1–20.

Atangana, A. (2015). On the stability and convergence of the time-fractional variable order telegraph equation. *Journal of Computational Physics*, *293*, 104–114.

Biazar, J., & Eslami, M. (2010). Analytic solution for Telegraph equation by differential transform method. *Physics Letters A*, *374*(29), 2904–2906.

Chen, J., Liu, F., & Anh, V. (2008). Analytical solution for the time-fractional telegraph equation by the method of separating variables. *Journal of Mathematical Analysis and Applications*, *338*(2), 1364–1377.

Das, S., Vishal, K., Gupta, P. K., & Yildirim, A. (2011). An approximate analytical solution of time-fractional telegraph equation. *Applied Mathematics and Computation*, *217*(18), 7405–7411.

Dehghan, M., & Ghesmati, A. (2010). Solution of the second-order one-dimensional hyperbolic telegraph equation by using the dual reciprocity boundary integral equation (DRBIE) method. *Engineering Analysis with Boundary Elements*, *34*(1), 51–59.

Dehghan, M., & Salehi, R. (2012). A method based on meshless approach for the numerical solution of the two - space dimensional hyperbolic telegraph equation. *Mathematical Methods in the Applied Sciences*, *35*(10), 1220–1233.

Dosti, M., & Nazemi, A. (2012). Quartic B-spline collocation method for solving one-dimensional hyperbolic telegraph equation. *Journal of Information and Computing Science*, *7*(2), 83–90.

Hosseini, V. R., Chen, W., & Avazzadeh, Z. (2014). Numerical solution of fractional telegraph equation by using radial basis functions. *Engineering Analysis with Boundary Elements*, *38*, 31–39.

Javidi, M., & Nyamoradi, N. (2013). Numerical solution of telegraph equation by using LT inversion technique. *International Journal of Advanced Mathematical Sciences*, *1*(2), 64–77.

Jiwari, R., Pandit, S., & Mittal, R. C. (2012). A differential quadrature algorithm for the numerical solution of the second-order one dimensional hyperbolic telegraph equation. *International Journal of Nonlinear Science*, *13*(3), 259–266.

Lakestani, M., & Saray, B. N. (2010). Numerical solution of telegraph equation using interpolating scaling functions. *Computers & Mathematics with Applications*, *60*(7), 1964–1972.

Mittal, R. C., & Jain, R. (2012). Redefined cubic B-splines collocation method for solving convection–

diffusion equations. *Applied Mathematical Modelling*, *36*(11), 5555–5573.
Phillips, G. M. (2003). *Interpolation and approximation by polynomials* (Vol. 14). Springer Science & Business Media.
PM, P. (1975). *Spline and variational methods*. Wiley, New York.
Rashidinia, J., & Jokar, M. (2016). Application of polynomial scaling functions for numerical solution of telegraph equation. *Applicable Analysis*, *95*(1), 105–123.
Saadatmandi, A., & Dehghan, M. (2010). Numerical solution of hyperbolic telegraph equation using the Chebyshev tau method. *Numerical Methods for Partial Differential Equations: An International Journal*, *26*(1), 239–252.
Sharifi, S., & Rashidinia, J. (2016). Numerical solution of hyperbolic telegraph equation by cubic B-spline collocation method. *Applied Mathematics and Computation*, *281*, 28–38.
Srinivasa, K., & Rezazadeh, H. (2021). Numerical solution for the fractional-order one-dimensional telegraph equation via wavelet technique. *International Journal of Nonlinear Sciences and Numerical Simulation*, *22*(6), 767–780.
Wang, K.-L., Yao, S.-W., Liu, Y.-P., & Zhang, L.-N. (2020). A fractal variational principle for the telegraph equation with fractal derivatives. *Fractals*, *28*(04), 2050058.