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Extracting new inflation model from the most general scalar-tensor theory

Extracting new inflation model from the most general scalar-tensor theory

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Abstract. Inflation is now the strongest scenario to be the candidate of early universe. Until recently, there are many models tried to give explanation about this phenomenon, one of them is called Horndeski theory, the most general scalar-tensor theory. In this paper, we investigate the no-ghost and Laplacian stability conditions for scalar and tensor perturbation for Horndeski theory, and obtains exactly the same conditions as the actual stability conditions from some particular gravity-coupled scalar field cosmological model. Therefore, there are big possibility to using the conditions obtained to become a constraint for extracting the new cosmological model for inflation scenario.

1. Introduction

The accepted viable phenomenological scenario describing the exponential expansion in the early time is the inflationary universe [1, 2] that in first place developed to become a possible solution to the horizon and flatness problems in cosmology. There was several model established to explain this phenomena [3, 4, 5, 6, 7], but until this time, inflation is still become a speculative scenario, although the observation of temperature fluctuation of the Cosmological Microwave Background (CMB) by WMAP [8] and COBE [9] bolster the slow-roll inflationary universe scenario driven by single scalar degree of freedom, because of the lack of knowledge about the origin of the scalar field.

Cosmological model where scalar field interact with curvature tensor via various interaction model have given favorable solutions for some problem in cosmology, such as for late-time acceleration [10], dark Matter and dark Energy [11], and inflation [12]. In 1974, Horndeski [13] derived the most general scalar-tensor theories, with lagrangian,

$$L = \sum_{i=2}^{5} L_i, \tag{1}$$

where, with $X = \partial_\mu\phi\partial^\mu\phi$,

$$L_2 = \mathcal{K}(\phi, X), \tag{2}$$
$$L_3 = -G_3(\phi, X)\Box\phi, \tag{3}$$
$$L_4 = G_4(\phi, X)R - 2G_{4X}(\phi, X)[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}], \tag{4}$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)(\phi_{;\mu\nu}\phi^{;\mu\nu}) + 2(\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma})], \tag{5}$$

with d'Alembertian or $\Box$ (square) operator is defined as $\Box = \partial^\mu\partial_\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$, where we consider natural unit, $c = 1$. This theory comprises all the models with gravity-coupled scalar field that previously studied by choosing particular coefficient functions $(\mathcal{K}(\phi, X), G_i(\phi, X))$, such as minimally coupled scalar field [14], Brans-Dicke theory [15], Dilaton gravity [16], $f(R)$ gravity [17], derivative couplings [18], Gauss-Bonnet couplings [6] and many others. Although the Horndeski theory is not fully satisfactory, this theory may indeed offer alluring solution for cosmology features with the possibility to obtain a new theory of inflation by adjusting the coefficient functions.

The organization of the paper is as follows, In section 2, we spell out the no-ghost and Laplacian stability condition of General Gravitational Theories from [19], then use it for Horndeski Lagrangian in Section 3. In Section 4, we try to find the stability conditions for some model (in this works for Brans-Dicke theory and Gauss-Bonnet couplings) and compare the result with actual stability conditions obtained before. The final chapter is for conclusion.

2. No-Ghost and Laplacian Stability of General Gravitational Theories

The ADM metric is given by,

$$ds^2 = g_{\mu\nu} = -N^2dt^2 + h_{ij}(dx^i + N^idt)(dx^j + N^jdt), \tag{6}$$

with N is lapse function, N^i is shift vector and h_{ij} is the three-dimensional metric. Therefore we can get $g_{\mu\nu} = -N^2 + N^iN_i, N_i, h_{ij}$. An orthogonal vector at constant t of hypersurface Σ_t is given by $n_\mu = -Nt_{;\mu} = (-N, 0, 0, 0)$, so that $n^\mu = (\frac{1}{N}, \frac{-N^i}{N})$, so this orthogonal vector is timelike, $n^\mu n_\mu = -1$. The three-dimensional metric on the hypersurface, $h_{\mu\nu}$, has relation with four-dimensional metric $g_{\mu\nu}$, $h_{\mu\nu} = g_{\mu\nu} + n^\mu n^\nu$, satisfies orthogonal relation $n^\mu h_{\mu\nu} = 0$.

Extrinsic curvature is defined by,

$$K_{\mu\nu} = h^\lambda_\mu n_{\nu;\lambda} = n_{\nu;\mu} + n_\mu\dot{n}_\nu, \tag{7}$$

where $n^\mu K_{\mu\nu} = 0$, that means $K_{\mu\nu}$ is a quantity on hypersurface. The Geometry of the hypersurface can be expressed by $\mathcal{R}_{\mu\nu} =^{(3)} R_{\mu\nu}$, as,

$$R = \mathcal{R} + K_{\mu\nu}K^{\mu\nu} - K^2 + 2(Kn^\mu - \dot{n}^\mu)_{;\mu}, \tag{8}$$

with some scalar quantity defined as,

$$K \equiv K^\mu_\mu;\ \mathcal{S} \equiv K_{\mu\nu}K^{\mu\nu};\ \mathcal{R} \equiv \mathcal{R}^\mu_\mu;\ \mathcal{Z} \equiv \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu};\ \mathcal{U} \equiv \mathcal{R}_{\mu\nu}K^{\mu\nu}. \tag{9}$$

Therefore lagrangian from general gravitational theory depends of this scalar can be written as,

$$S = \int d^4x\sqrt{-g}L(N, K, \mathcal{S}, \mathcal{R}, \mathcal{Z}, \mathcal{U}; t). \tag{10}$$

If the Lagrangian above is expanded to second order, we can get,

$$L = \bar{L} + L_N\delta N + L_K\delta K + L_{\mathcal{S}}\delta\mathcal{S} + L_{\mathcal{R}}\delta\mathcal{R} + L_{\mathcal{Z}}\delta\mathcal{Z} + L_{\mathcal{U}}\delta\mathcal{U} + \frac{1}{2}(\delta N\frac{\partial}{\partial N} + \delta K\frac{\partial}{\partial K} + \delta\mathcal{S}\frac{\partial}{\partial\mathcal{S}} + \delta\mathcal{R}\frac{\partial}{\partial\mathcal{R}} + \delta\mathcal{Z}\frac{\partial}{\partial\mathcal{Z}} + \delta\mathcal{U}\frac{\partial}{\partial\mathcal{U}})^2 L. \quad (11)$$

where bar denotes the background quantity. To get the no-ghost and Laplacian stability, we consider the second-order of Lagrangian (11), then if we consider flat homogenous and isotropic (FLRW) metric

$$ds^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j, \quad (12)$$

as background metric, using relation,

$$\sqrt{-g} = \sqrt{\bar{h}} + \sqrt{\bar{h}}\delta N + \delta\sqrt{h} + \delta N\delta\sqrt{h} = a^3 + a^3\delta N + \delta\sqrt{h} + \delta N\delta\sqrt{h}, \quad (13)$$

the second order Lagrangian density can be obtained as,

$$\begin{aligned}\mathcal{L}_2 &= a^3L_2 + a^3\delta NL_1 + \delta N\delta\sqrt{h}L_0 + L_1\delta\sqrt{h}\\ &= \delta\sqrt{h}[(\dot{\mathcal{F}} + L_N)\delta N + \mathcal{E}\delta_1\mathcal{R}] + a^3[(L_N + \frac{1}{2}L_{NN})\delta N^2 + \mathcal{E}\delta_2\mathcal{R} + \frac{1}{2}\mathcal{A}\delta K^2\\ &+ \mathcal{B}\delta K\delta N + \mathcal{C}\delta K\delta_1\mathcal{R} + (\mathcal{D}+\mathcal{E})\delta N\delta_1\mathcal{R} + \mathcal{E}\delta_2\mathcal{R} + \frac{1}{2}\mathcal{G}\delta_1\mathcal{R}^2 + L_{\mathcal{S}}\delta K^\mu_\nu\delta K^\nu_\mu + L_{\mathcal{Z}}\delta\mathcal{R}^\mu_\nu\delta\mathcal{R}^\nu_\mu].\end{aligned} \quad (14)$$

with,

$$\begin{aligned}&\mathcal{A} = L_{KK} + 4H^2L_{\mathcal{SS}} + 4HL_{\mathcal{S}K},\ \mathcal{B} = L_{KN} + 2HL_{\mathcal{S}N},\\ &\mathcal{C} = L_{K\mathcal{R}} + 2HL_{\mathcal{SR}} + \frac{1}{2}L_{\mathcal{U}} + HL_{K\mathcal{U}} + 2H^2L_{\mathcal{SU}},\ \mathcal{D} = L_{N\mathcal{R}} - \frac{1}{2}\dot{L}_{\mathcal{U}} + HL_{N\mathcal{U}},\\ &\mathcal{E} = L_{\mathcal{R}} + \frac{1}{2}\dot{L}_{\mathcal{U}} + \frac{3}{2}HL_{\mathcal{U}},\ \mathcal{F} = L_K + 2HL_{\mathcal{S}},\ \mathcal{G} = L_{\mathcal{RR}} + 2HL_{\mathcal{RU}} + H^2L_{\mathcal{UU}}.\end{aligned} \quad (15)$$

and $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter and dot denotes a derivative with respect to time.

2.1. Scalar Perturbation

For scalar pertubation, we take the gauge $h_{ij} = a^2e^{2\zeta}\delta_{ij}$, then the second-order Lagrangian density (14) for scalar perturbation can be obtained as,

$$\begin{aligned}\mathcal{L}_2^{(s)} &= a^3\{\frac{1}{2}(2L_N + L_{NN} + 9H^2\mathcal{A} - 6H\mathcal{B} + 6H^2L_{\mathcal{S}})\delta N^2\\ &+ [\mathcal{W}(3\dot{\zeta} - \frac{\partial^2\psi}{a^2}) + 4(3H\mathcal{C} - (\mathcal{D}+\mathcal{E}))\frac{\partial^2\zeta}{a^2}]\delta N - (3\mathcal{A} + 2L_{\mathcal{S}})\dot{\zeta}\frac{\partial^2\psi}{a^2} - 12\mathcal{C}\dot{\zeta}\frac{\partial^2\zeta}{a^2}\\ &+ (\frac{9}{2}\mathcal{A} + 3L_{\mathcal{S}})\dot{\zeta}^2 + 2\mathcal{E}\frac{(\partial^2\zeta)^2}{a^4} + \frac{1}{2}(\mathcal{A} + 2L_{\mathcal{S}})\frac{(\partial^2\psi)^2}{a^4} + 4\mathcal{C}\frac{(\partial^2\psi)(\partial^2\zeta)}{a^4} + 2(4\mathcal{G} + 3L_{\mathcal{Z}})\frac{(\partial^2\zeta)^2}{a^4}\}.\end{aligned} \quad (16)$$

Variations of the second-order action $S_2^{(s)} = \int d^4x\mathcal{L}_2^{(s)}$ with respect to δN and $\partial^2\psi$ lead to,

$$(2L_N + L_{NN} + 9H^2\mathcal{A} - 6H\mathcal{B} + 6H^2L_{\mathcal{S}})\delta N + \mathcal{W}(3\dot{\zeta} - \frac{\partial^2\psi}{a^2}) + 4(3H\mathcal{C} - (\mathcal{D}+\mathcal{E}))\frac{\partial^2\zeta}{a^2} = 0, \quad (17)$$

$$\mathcal{W}\delta N + (3\mathcal{A} + 2L_{\mathcal{S}})\dot{\zeta} - (\mathcal{A} + 2L_{\mathcal{S}})\frac{(\partial^2\psi)}{a^2} - 4\mathcal{C}\frac{(\partial^2\zeta)}{a^2} = 0. \quad (18)$$

There exist high-order spatial derivatives, so impose conditions,

$$\mathcal{A} + 2L_{\mathcal{S}} = 0;\ \mathcal{C} = 0;\ 4\mathcal{G} + 3L_{\mathcal{Z}} = 0, \tag{19}$$

to get rid of this high-order spatial derivatives. Using this conditions, The second-order Lagrangian density for scalar perturbation (16) can be written as,

$$\mathcal{L}_2^{(s)} = a^3\mathcal{Q}_s[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial\zeta)^2]; \tag{20}$$

$$Q_s \equiv \frac{2L_{\mathcal{S}}[3\mathcal{B}^2 + 4L_{\mathcal{S}}(2L_N + L_{NN})]}{\mathcal{W}^2};\ c_s^2 \equiv \frac{2}{\mathcal{Q}_s}(\dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E}). \tag{21}$$

Therefore, to avoid the existence of ghost and Laplacian instability, these conditions must be satisfied,

$$\mathcal{Q}_s > 0\ \rightarrow \frac{2L_{\mathcal{S}}[3\mathcal{B}^2 + 4L_{\mathcal{S}}(2L_N + L_{NN})]}{\mathcal{W}^2} > 0 \rightarrow 9\mathcal{W}^2 + 8L_{\mathcal{S}}\omega > 0 \tag{22}$$

$$c_s^2 > 0\ \rightarrow \dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E} > 0, \tag{23}$$

with,

$$\mathcal{W} = L_{KN} + 2HL_{\mathcal{S}N} + 4HL_{\mathcal{S}};\ \omega = 3L_N + \frac{3}{2}L_{NN} - 9H(L_{KN} + 2HL_{\mathcal{S}N} - 18H^2L_{\mathcal{S}}). \tag{24}$$

$$\mathcal{M} \equiv \frac{4L_{\mathcal{S}}(\mathcal{D} + \mathcal{E})}{\mathcal{W}}. \tag{25}$$

2.2. Tensor Perturbation

For tensor perturbation, we take the three-dimensional metric including tensor perturbation γ_{ij},

$$h_{ij} = a^2(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{il}\gamma^{lj});\ \gamma_{ii} = \partial_i\gamma_{ij} = 0. \tag{26}$$

From equation (14) the second-order Lagrangian density for tensor perturbation can be obtained as,

$$\mathcal{L}_2^{(h)} = \frac{a^3}{4}\mathcal{Q}_t[\dot{\gamma_{ij}}^2 - \frac{c_t^2}{a^2}(\partial_k\gamma_{ij})^2];\ \mathcal{Q}_t \equiv L_{\mathcal{S}};\ c_t^2 \equiv \frac{\mathcal{E}}{L_{\mathcal{S}}}. \tag{27}$$

Therefore, to avoid the existence of ghost and Laplacian instability, these conditions must be satisfied,

$$\mathcal{Q}_t > 0\ \rightarrow L_{\mathcal{S}} > 0;\ c_t^2 > 0\ \rightarrow \mathcal{E} > 0. \tag{28}$$

3. Horndeski Theory in ADM Variable

To get the condition to avoid the existence of ghost and Laplacian instability for Horndeski Lagrangian, firstly we write the Horndeski Lagrangian in ADM variable. Using the way we through in section 2, we can obtain the total Lagrangian for Horndeski theory in ADM variable as,

$$L = \mathcal{K} + 2(-X)^{\frac{3}{2}}F_{3X}K - F_{3\phi}X + G_4\mathcal{R} + (2XG_{4X} - G_4)(K^2 - \mathcal{S}) - 2\sqrt{-X}G_{4\phi}K + \sqrt{-X}[F_5(\frac{1}{2}K\mathcal{R} - \mathcal{U}) - G_{5X}(2H^2 - 2KH + K^2 - \mathcal{S})] + \frac{1}{2}[(G_{5\phi} - F_{5\phi})\mathcal{R} + G_{5\phi}(K^2 - \mathcal{S})], \tag{29}$$

where we take,

$$G_3 = F_3 + 2XF_{3X};\ G_{5X} = F_5/2X + F_{5X}, \tag{30}$$

with $X = \partial_\mu\phi\partial^\mu\phi = -\dot{\phi}^2$, $G_{iX} \equiv \partial G_i/\partial X$ and $G_{i\phi} \equiv \partial G_i/\partial\phi$.

Therefore we can obtain the conditions to avoid the existence of ghost and Laplacian Instability for Horndeski theory as, for tensor perturbation,

$$L_{\mathcal{S}} = G_4 - 2XG_{4X} - H\dot{\phi}XG_{5X} - \frac{1}{2}XG_{5\phi} > 0, \tag{31}$$

$$\mathcal{E} = G_4 + \frac{1}{2}XG_{5\phi} - XG_{5X}\ddot{\phi} > 0 \tag{32}$$

and for scalar perturbation, is same as equation (22)-(23).

4. The Stability Condition for Some Particular Gravity-Coupled Scalar Field Cosmological Model

Now we try to using this condition to get the stability conditions for an example model that can be obtained from choosing the K, G_3, G_4, G_5 functions. In this works we try to find the stability condition for two cosmological model, The Brans-Dicke theory and Gauss-Bonnet coupling.

4.1. The Brans-Dicke (BD) Theory

The Lagrangian of Brans-Dicke theory is given by,

$$K = -\frac{M_{pl}\omega_{BD}X}{2\phi} - V(\phi);\ G_3 = 0;\ G_4 = \frac{1}{2}M_{pl};\ G_5 = 0. \tag{33}$$

There are several model that equivalent to BD Theory such as $f(R)$ gravity that equivalent to BD theory with $\omega_{BD} = 0$ based on the choice of the coefficient function and dilaton gravity that correspond to $\omega = -1$. For this model, the conditions (31) and (32) are satisfied for

$$\phi > 0. \tag{34}$$

Other than that, the value of (22) for this model can be written as,

$$\dot{\phi}^2(9 + 6\omega_{BD}) > 0. \tag{35}$$

Because $\dot{\phi}^2$ always have positive quantity (real scalar field), so for BD theory, we can get the no-scalar-ghost condition,

$$\omega_{BD} > -3/2. \tag{36}$$

Therefore, we can say that some model which equivalent with BD theory mentioned before, satisfied the no-scalar-ghost condition.

4.2. Gauss-Bonnet Coupling

There are several works with Gauss-Bonnet term [6, 7] and Hikmawan, et. al. [20] found that this model is stable for tensor perturbation if the squared effective speed of sound is positive,

$$c^2_{sound} \equiv \frac{1 + \ddot{f}}{1 + H\dot{f}} \backsimeq \frac{\ddot{f}}{H\dot{f}} = \frac{f^{(1)}\ddot{\phi} + f^{(2)}\dot{\phi}^2}{H\dot{\phi}f^{(1)}} > 0;\ f^{(n)} \equiv \partial^n f(\phi)/\partial\phi^n, \tag{37}$$

where we get the second relation when inflation occurs, the Ricci(gravity) term is dominated by the Gauss-Bonnet term, so the Ricci (gravity) term can be neglected [6, 7]. The cosmological model with Gauss-Bonnet term can be obtained from Horndeski theory by the choice [21],

$$\mathcal{K} = -2f^{(4)}X^2[3 - \ln(-X/2)];\; G_3 = 2f^{(3)}X[7 - 3\ln(-X/2)],$$
$$G_4 = 2f^{(2)}X[2 - \ln(-X/2)];\; G_5 = 4f^{(1)}\ln(-X/2), \tag{38}$$

then using equation (31) and (32) we can get the exactly same stability condition for tensor perturbation,

$$c_t^2 \equiv \frac{\mathcal{E}}{L_{\mathcal{S}}} = \frac{\ddot{\phi}f^{(1)} - Xf^{(2)}}{H\dot{\phi}f^{(1)}} = \frac{\ddot{\phi}f^{(1)} + \dot{\phi}^2 f^{(2)}}{H\dot{\phi}f^{(1)}} > 0. \tag{39}$$

5. Conclusion

We have analyzed an obtained the no-ghost and Laplacian stability conditions of general gravitational theories and then implementing the general form to obtain the no-ghost and Laplacian stability conditions for Horndeski Theory. Using the conditions obtained, we try to find the stability conditions for some particular gravity-coupled scalar field cosmological model (BD theory and Gauss-Bonnet coupling) and found exactly the same conditions as the actual stability conditions from the models. As indicated above, one can find the new and more realistic cosmological model by adjusting the coefficient function $(K(\phi, X), G_i(\phi, X))$ in Horndeski Lagrangian. Therefore, there are big possibility to using the no-go and Laplacian stability conditions obtained above to become a constraint for extracting the new cosmological model, for inflation solution, or for the other big problems in cosmology. We leave this excitement to become motivation of future works.

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