Field Localization for the 5-Dimensional Nonminimal Derivative Coupling of The Scalar Field with Modified Randall-Sundrum Model

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Field Localization for the 5-Dimensional Nonminimal Derivative Coupling of The Scalar Field with Modified Randall-Sundrum Model

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Abstract. Among the extra dimension models, Randall-Sundrum(RS) model is one of the most famous examples for its well--addressing of the hierarchy problems. However, the study [1] has introduced the modified RS model which more superior due to its better localization for scalar, vector, and spinor fields. This study uses five dimensional Einstein tensor coupled scalar field or Non Minimal Derivative Coupling(NMDC) of the scalar field, such model has been studied widely for GR and cosmological purpose. Here, we study the localization properties of NMDC model of scalar field using the modified RS model. For the warp factor k is equal to zero, we find that five-dimensional NMDC of the scalar field is localized in the brane as minimal coupling.

1. Introduction

The extra-dimensional models have been studied thoroughly in general relativity recently. One of those is Randal-Sundrum(RS) model, which well known for its well addressing of Higgs hierarchy problems [2]. Considering the particle physics problems in extra dimension model, one can verify whether the ordinary matter is to be confined in four-dimensional spacetime, brane, that is embedded in the higher dimension, bulk [1]. Mathematically, the condition for confined matter or localized matter is decomposing the Hilbert action integral into the extra dimension part and the four dimensional part. Therefore, the extra dimension integral for all its space must be finite so the Hilbert action becomes four-dimensional action. However, the study [1] showed massive scalar fields are not localized for the RS model, hence the modified RS model was introduced by [1] and find that the scalar field is localized, also need to be mentioned that unlocalising massless and massive spinor fields on the brane also proved by [1]. Many models have been designed for analyzing the matter to gravity and the theory which generalized matter to gravity is known as nonminimal derivative couplings(NMDC) model of scalar field which introduced by [3]. The model has been used widely for cosmological and general relativity applications. Analyzing cosmological aspect using NMDC model of scalar fields in five dimensions has been studied by [4], extra dimensional stability analysis of NMDC model has been studied in [5], and some related works about NMDC model in extra dimension also shown in [6, 7, 8]. In this work, the localization for five dimensional NMDC model of the scalar field is investigated using modified RS model. Begin at considering lagrangian for five-dimensional NMDC of scalar fields then putting it into the Hilbert action and analyze the localization properties using the decomposed integrals.

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2. Equation of Motion for Nonminimal Derivative Couplings

This work used Lagrangian for NMDC of the scalar field in five dimensions[4] in the following form

$$L = g^{AB}\partial_A\Phi\partial_B\Phi + \xi G^{AB}\partial_A\Phi\partial_B\Phi, \tag{1}$$

with a scalar field $\Phi(r, x^\mu) = \phi(x^\mu)\chi(r)$. The extra dimension denoted by *r*, g^{AB} is the metric, G^{AB} is Einstein tensor, and ξ is the coupling constant. The dummy indexes *A* and *B* are five-dimensional components. Taking Euler-Lagrange equation for the equation (1) and one can obtain

$$\frac{1}{\phi}\left[\sqrt{-g}\left(g^{\mu\nu} + \xi G^{\mu\nu}\right)\right]\partial_\mu\partial_\nu\phi + \frac{1}{\chi}\partial_5\left[\sqrt{-g}\left(g^{55}\partial_5\chi + \xi G^{55}\partial_5\chi\right)\right] = 0. \tag{2}$$

The modified RS model which proposed by [1] taking the form

$$ds^2 = e^{-2k|r|}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dr^2\right). \tag{3}$$

with *k* is the warp factor and for the convenient take $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. Thus, the equation (2) becomes a differential equation given by

$$\frac{1}{\phi}\alpha^{\mu\nu}\partial_\mu\partial_\nu\phi + \frac{1}{\chi}\beta(r)\partial_5\chi + \frac{1}{\chi}\gamma(r)\partial_5\partial_5\chi = 0, \tag{4}$$

where $\alpha^{\mu\nu} \equiv \sqrt{-g}\left(g^{\mu\nu} + \xi G^{\mu\nu}\right)$, $\beta(r) \equiv -3ke^{-3k|r|} - \xi 6k^3 e^{-k|r|}$, and $\gamma(r) \equiv e^{-3k|r|} + \xi 6k^2 e^{-k|r|}$. Using separation variables properties, from equation (4) one can get the equation of motion for the extra dimension part in a form

$$\left(-3ke^{-3k|r|} - \xi 6k^3 e^{-k|r|}\right)\partial_5\chi + \left(e^{-3k|r|} + \xi 6k^2 e^{-k|r|}\right)\partial_5\partial_5\chi = m^2\chi \tag{5}$$

and the four-dimensional part as

$$\alpha^{\mu\nu}\partial_\mu\partial_\nu\phi = -m^2\phi. \tag{6}$$

with its mass $m^2 = \omega^2 - \kappa^2$.

3. The Solution of The Equation of Motion

We need to find the solutions of the equations (4) and (5) for examine the localization properties of the NMDC model of a scalar field. The equation (5) seems to be too difficult to find its solutions analytically. However, the first attempt for finding its solution is done in this study. The condition for this special case is taking the warp factor *k* equal to zero. Furthermore, the solutions are

$$\phi(x^\mu) = Ae^{\kappa_\mu x^\mu}, \tag{7}$$

and for the extra-dimensional taking the form

$$\chi(r) = Ce^{mr} + De^{-mr}. \tag{8}$$

As the distance *r* more further from the brane one expected the field $\chi(r)$is zero asymptotically. So, the complete solution is given by

$$\Phi(r, x^\mu) = \phi(x^\mu)\chi(r) = Ae^{\kappa_\mu x^\mu - mr}. \tag{9}$$

The five-dimensional Hilbert action form of the Lagrangian (1) is given by

$$S_5 = \int\sqrt{-g}\left(g^{AB}\partial_A\phi\partial_B\phi + \xi G^{AB}\partial_A\phi\partial_B\phi\right)d^5x, \tag{10}$$

the integral above can be decomposed to be two parts, it is the definite integral of extra dimension part multiplied by its four-dimensional components, so the integral may be expressed as

$$S_5 = \int_0^\infty g^{\mu\nu}\sqrt{-g}\chi^2 dr \int \partial_\mu\phi\partial_\nu\phi\, d^4x + \int_0^\infty g^{55}\sqrt{-g}\partial_5\chi\partial_5\chi\, dr \int \phi^2\, d^4x$$
$$+\xi\int_0^\infty G^{\mu\nu}\sqrt{-g}\chi^2\, dr \int \partial_\mu\phi\partial_\nu\phi\, d^4x + \xi\int_0^\infty G^{55}\sqrt{-g}\partial_5\chi\partial_5\chi\, dr \int \phi^2\, d^4x. \qquad (11)$$

The localization means the definite integral of extra dimension parts has to be constants. Hence, the integral becomes

$$\int_0^\infty g^{\mu\nu}\sqrt{-g}\chi^2\, dr = \eta^{\mu\nu}M,\ \int_0^\infty G^{\mu\nu}\sqrt{-g}\chi^2\, dr = \eta^{\mu\nu}N, \qquad (12)$$

$$\int_0^\infty g^{55}\sqrt{-g}\partial_5\chi\partial_5\chi\, dr = c^2,\ \int_0^\infty G^{55}\sqrt{-g}\partial_5\chi\partial_5\chi\, dr = d^2. \qquad (13)$$

Thus, the action (10) can be expressed

$$S_5 = \int\left(\eta^{\mu\nu}(M+\xi N)\partial_\mu\phi\partial_\nu\phi + \left(c^2+\xi d^2\right)\phi^2\right)d^4x. \qquad (14)$$

By putting the solution (9) into equation (12) and (13) one can find the constants $N = \frac{D}{2m}$, $M = 0$, $c^2 = \frac{Dm}{2}$, and $d^2 = 0$. Finally, the action taking the form

$$S_5 = \int\left(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)d^4x. \qquad (15)$$

The minimal coupling equation (15) indicates the NMDC model of scalar field using RS metric is localized for the case warp factor is equal to zero and interpreted as the five-dimensional scalar field is confined in the brane.

4. Conclusions

This study verified whether the scalar fields is confined in the brane which is embedded in a bulk or not by considering its localizations properties. The Lagrangian for NMDC of scalar fields as the general matter to gravity is applied to the five-dimensional action and decomposing the integrals. The localization condition is the integral of each extra dimension parts should be constant. So, the integral can be written by equation (12) an (13). We find that localization of scalar field of NMDC model is occurred by taking the warp factor *k* is equal to zero as indicated by (15). In this case, the localization properties reducts nonminimal derivative coupling into minimal coupling by vanishing the coupling term. The first attempt above is a simplification of finding the solutions of (5), the equation can be solved numerically with some initial conditions so the localizations properties can be conceived intensely.

5. References

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