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Investigating Starobinsky's Inflation Model Using Arnowitt-Deser-Misner (ADM) Formalism for Scalar Perturbation in The Cosmic Inflation Epoch

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Abstract. In this paper, we investigate Starobinsky's inflation model for scalar perturbation during cosmic inflation epoch by using Arnowitt-Deser-Misner (ADM) formalism. We implemented this formalism to the action of curvature-scalar of Starobinsky's model, which is one of the models of cosmic inflation. As the result of this investigation, we found the power spectrum of scalar perturbation in the form of Starobinsky's inflation model which has a linear correlation to the comoving Hubble radius which is shrinking during inflation epoch.

Keyword. Inflation, Starobinsky's inflation model, Scalar Perturbation, ADM Formalism, Cosmology

1. Introduction

The correction of the photons's temperature in the cosmic microwave background (CMB) and the formation of large-scale structures observed today is believed come from a quantum fluctuation that growth very rapidly during inflation epoch. These perturbations include perturbation in metric and also perturbation in the matter. Perturbation of the metric can be a scalar perturbation, a vector perturbation



or a tensor perturbation. The equations of motion of the perturbation can be derived by Einstein's equation or through action approach.

The application of Arnowitt-Deser-Misner (ADM) metric has been widely used to describe the dynamics of quantum fluctuation. ADM formalism is a way to describe space-time into the slices of hypersurface for a constant time. Inflation requires a negative pressure to produce the accelerated expansion. Therefore, constituent that generate inflation is not the type of ordinary matter that exists today [1-4]. There are a lot of models of inflation which has been filed with the different theoretical motivation and different predictions of observations.

One of the best way to describe the interaction of a gravitational field that is using the principle of action. The action of each perturbation can be expanded to any order. Since the perturbation is a quantum fluctuation, a quantization of classical observable to the quantum observable is needed. The equation of motion which was expanded up to second order has a form that similar to the form of a harmonic oscillator's equation of motion. Therefore, the quantization process of scalar perturbation can be done in the same way with harmonic oscillator by using the creation and annihilation operator [1, 5].

Statistical properties of scalar perturbation can be determined by deriving the two-point correlation function or the power spectrum where it is the Fourier transform of the correlation function of the perturbation. Two-point correlation function can be derived by calculating the probability amplitude of each perturbation [1, 2, 6, 7].

In this work, we modify the parts of scalar curvature by implementing the Starobinsky's inflation model [8, 9]. We interested to investigate the cosmological perturbation in term of the scalar perturbation by using modify scalar curvature action for Starobinsky's model and ADM formalism during cosmic inflation epoch.

2. Arnowitt-Deser-Misner (ADM) Formalism

The Arnowitt-Deser-Misner (ADM) formalism describes space-time by dividing the space-time into the slices of space with a constant time.

$$ds^2 = -(N^2 - N^i N_i) dt^2 + 2N_i dx^i dt + h_{ij} dx^i dx^j. \quad (1)$$

The ADM formalism was implemented by replacing the variables in the action with ADM variables. The most common action to describe the dynamics of space-time is the Einstein-Hilbert action as follow

$$S = \frac{1}{2} \int d^4x \sqrt{-g} R. \quad (2)$$

After some integral by part was done to the equation (2), the Einstein-Hilbert action in term of ADM formalism would be obtained,

$$S = \frac{1}{2} \int d^4x \sqrt{h} N \left[R^{(3)} - K_i^j K_j^i + K_{ij} K^{ij} \right], \quad (3)$$

where $K_{ij} = \frac{1}{2N} (\nabla_i^{(3)} N_j + \nabla_j^{(3)} N_i - \dot{h}_{ij})$ is the extrinsic curvature.

3. ADM Formalism for Scalar Field Inflation

The action of scalar field form,

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [R - (\nabla\Phi)^2 - 2V(\Phi)]. \quad (4)$$

The action of scalar field in the ADM formalism is:

$$S = \frac{1}{2} \int d^4x \sqrt{h} N \left[R^{(3)} + \frac{1}{N^2} (E_{ij} E^{ij} - E^2) + \frac{1}{N^2} (\dot{\Phi} - N^i \partial_i \Phi)^2 - h^{ij} \partial_i \Phi \partial_j \Phi - 2V(\Phi) \right], \quad (5)$$

where $E_{ij} \equiv -NK_{ij}$. A hamiltonian density of a system defined by

$$\mathcal{H} = \pi^{ij} \dot{h}_{ij} - \mathcal{L}, \quad (6)$$

where π^{ij} is conjugate momentum. Therefore the Hamiltonian density of the action of scalar field is

$$\begin{aligned} \mathcal{H} = & 2\pi^{ij} E_{ij} - 2N_j \nabla_i \left(h^{-\frac{1}{2}} \pi^{ij} \right) \\ & - \sqrt{h} N \left[R^{(3)} + \frac{1}{N^2} (E_{ij} E^{ij} - E^2) + \frac{1}{N^2} (\dot{\Phi} - N^i \partial_i \Phi)^2 - h^{ij} \partial_i \Phi \partial_j \Phi \right. \\ & \left. - 2V(\Phi) \right]. \end{aligned} \quad (7)$$

Then we have two constraint equation: the Hamiltonian constraint (which is given by derivation of Hamiltonian with respect to the lapse function),

$$R^{(3)} - \frac{1}{N^2} (E_{ij} E^{ij} - E^2) - \frac{1}{N^2} (\dot{\Phi})^2 - 2V(\Phi) = 0. \quad (8)$$

and the Gauss constraint (which is given by derivation of Hamiltonian with respect to the shift vector),

$$\nabla_i \left[\frac{1}{N} (h_{il} E^{lj} - \delta_l^j E) \right] = 0. \quad (9)$$

To obtain solution of constrains equation, we use perturbation of lapse function and shift vector and also we use the Maldacena gauge, respectively

$$N \equiv 1 + \alpha_1 + \dots, \text{ and } N_i \equiv \partial_i \psi + \tilde{N}_i^{(1)} + \dots \quad (10)$$

$$h_{ij} = a^2 (e^{2\zeta} \delta_{ij}) \quad (11)$$

The solution of Hamiltonian constraint at zeroth order is

$$H^2 = \frac{1}{6} \dot{\Phi}^2 + \frac{1}{3} V(\Phi), \quad (12)$$

which is the Friedmann equation. For Gauss constraint, the zeroth order solution is zero. Meanwhile, the lapse function and shift vector at the first order respectively are

$$\alpha_1 = \frac{\dot{\zeta}}{H}, \text{ and } \psi^{(1)} = \frac{1}{2} a^2 \frac{\dot{\Phi}^2}{H^2} \partial^{-2} \zeta - \frac{1}{H} \zeta. \quad (13)$$

Then, the second order of scalar-tensor action in the ADM formalism is

$$S = \frac{1}{2} \int d^4x a^3 \frac{\dot{\Phi}^2}{H^2} [\zeta^2 - a^{-2} (\partial \zeta)^2]. \quad (14)$$

Power spectrum of scalar perturbation is

$$P_\zeta(k) = \frac{H^2}{\dot{\Phi}^2} \frac{H^2}{2k^3}. \quad (15)$$

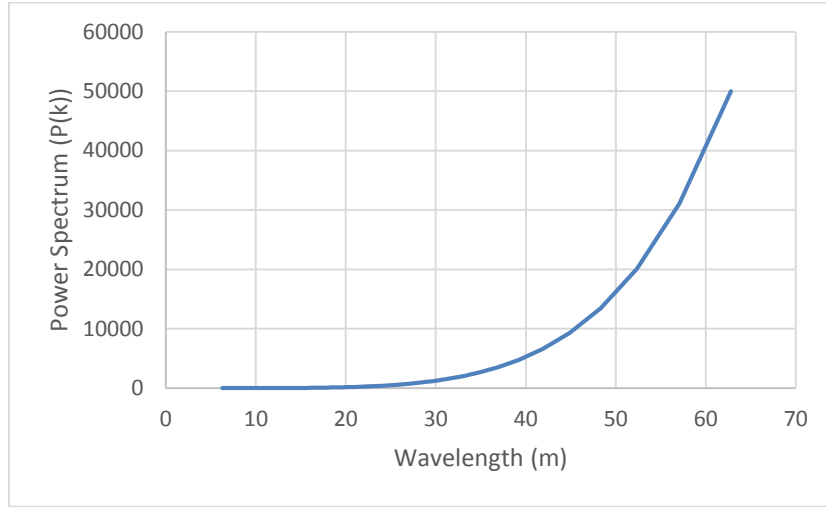


Figure 1. Plot Power Spectrum of Scalar Perturbation of Scalar Field Inflation Model to the Wavelength Perturbation

The power spectrum of scalar perturbation of scalar field inflation's model grows exponentially with respect to the wavelength of perturbation.

4. ADM Formalism for Starobinsky's Inflation

One of the simplest theory of modified gravity is the $f(R)$ theory. Starobinsky's gravity model is a modified model of gravity by adding a factor of R^2 in the action of the general scalar curvature action. The addition of R^2 factor is to provide a higher correction in the Einstein-Hilbert action [8-11]. The action of scalar curvature in the Starobinsky's model is

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[R + \frac{R^2}{6M^2} \right]. \quad (16)$$

The action of Starobinsky's model in the ADM formalism is:

$$S = \frac{1}{2} \int d^4x \sqrt{h} N \left[R^{(3)} + \frac{R^{(3)2}}{6M^2} + \frac{1}{N^2} (E_{ij} E^{ij} - E^2) + \frac{1}{3M^2 N^2} R^{(3)} (E_{ij} E^{ij} - E^2) + \frac{1}{6M^2 N^4} (E_{ij} E^{ij} - E^2)^2 \right]. \quad (17)$$

Using the same procedures by scalar field action, we get the Hamiltonian density of the action of Starobinsky's model is

$$\begin{aligned} \mathcal{H} = & 2\pi^{ij} E_{ij} - 2N_j \nabla_i \left(h^{-\frac{1}{2}} \pi^{ij} \right) \\ & - \frac{1}{2} \sqrt{h} N \left[R^{(3)} + \frac{R^{(3)2}}{6M^2} + \frac{1}{N^2} (E_{ij} E^{ij} - E^2) + \frac{1}{3M^2 N^2} R^{(3)} (E_{ij} E^{ij} - E^2) \right. \\ & \left. + \frac{1}{6M^2 N^4} (E_{ij} E^{ij} - E^2)^2 \right]. \end{aligned} \quad (18)$$

Then the Hamiltonian constraint is

$$\begin{aligned} & R^{(3)} + \frac{R^{(3)2}}{6M^2} - \frac{1}{N^2} (E_{ij} E^{ij} - E^2) - \frac{1}{3M^2 N^2} R^{(3)} (E_{ij} E^{ij} - E^2) - \frac{1}{2M^2 N^4} (E_{ij} E^{ij} - E^2)^2 \\ & = 0, \end{aligned} \quad (19)$$

and the Gauss constraint is

$$\nabla_i \left\{ \frac{1}{N} (h_{il} E^{ij} - \delta_l^j E) + \frac{1}{3M^2 N} R^{(3)} (h_{il} E^{ij} - \delta_l^j E) + \frac{1}{3M^2 N^3} (E_{ij} E^{ij} - E^2) (h_{il} E^{ij} - \delta_l^j E) \right\} = 0. \quad (20)$$

The solution of Hamiltonian constraint at zeroth order is

$$H^2 = \frac{M^2}{3}. \quad (21)$$

The solution in the equation (21) describe that the magnitude of H is constant. Based on the magnitude of H , it is indicated that $\frac{dH}{dt} = 0$. This condition agree with the inflation condition that $\frac{dH}{dt} = 0$. For Gauss constraint, the zeroth order solution is zero. Based on the solution of Gauss constraint and Hamiltonian constraint at zeroth order, we generated the first order perturbation of lapse function and shift vector, respectively

$$\alpha_1 = \frac{\dot{\zeta}}{H} + \frac{8}{9H^2 a^2} \partial_i \partial_i \zeta, \quad \text{and} \quad \psi^{(1)} = -\frac{1}{H} \zeta. \quad (22)$$

Finally, the action of scalar-curvature of order, is Starobinsky's model in the ADM formalism is expanded to second is

$$S = \frac{1}{2} \int d^4x \left[-\frac{4a^2}{3} (\partial_i \zeta)^2 + 18H^2 a^4 \zeta^2 \right]. \quad (23)$$

The equation of motion of scalar perturbation was derived directly by using Euler-Lagrange equation,

$$\frac{27}{2} H^2 a^2 \zeta + \partial_i \partial_i \zeta = 0. \quad (24)$$

One can also get the power spectrum of scalar perturbation,

$$P_\zeta(k) = \sqrt{\frac{1}{54} \frac{\hbar}{aH} \frac{(2\pi)^3}{k}}. \quad (25)$$

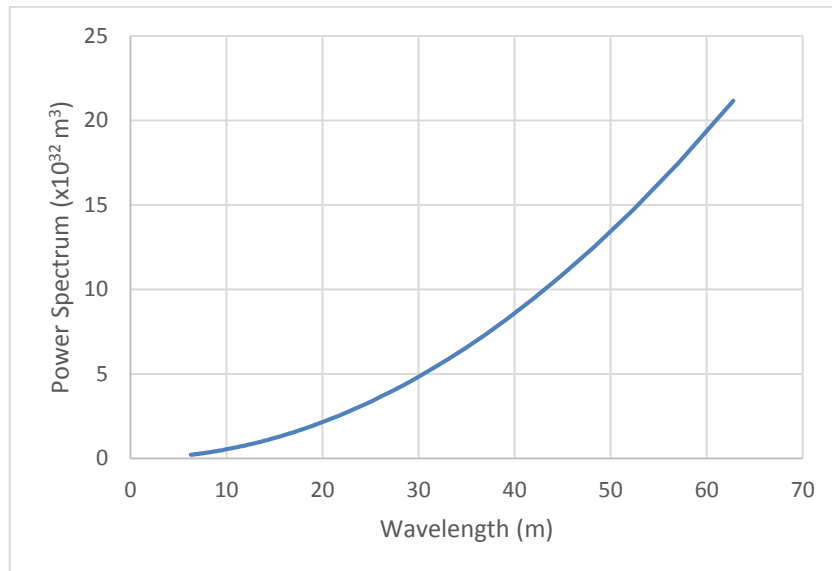


Figure 2. Plot Power Spectrum of Scalar Perturbation of Starobinsky's Inflation Model to The Wavelength Perturbation.

The power spectrum of scalar perturbation of Starobinsky inflation's model grow exponentially respect to the wavelength of perturbation.

5. Conclusion

Using the same procedures of scalar field inflation, ADM formalism is implement to the action of Starobinsky's model expanded to the second order action. The equation of motion of inflationary Starobinsky's model differs from equation of motion of scalar field because there were different approach for each model. Scalar field inflation model implemented modified matter approach, while for the Starobinsky's inflation model implemented modified gravity approach. In the Starobinsky's inflation model, the equation of motion of scalar perturbation had the same form of harmonic oscillator equation of motion. Meanwhile, in the scalar field inflation, the equation of motion of scalar perturbation had the same form of spherical Bessel function at first order.

Furthermore, the power spectrum of scalar perturbation of inflationary Starobinsky's model is analyzed. It had linier correlation to the commoving Hubble radius $\left(\frac{1}{aH}\right)$ with $\sqrt{\frac{1}{54}}$ as an additional factor. During cosmic inflation epoch, comoving Hubble radius shrink until at the end of inflation. After the inflationary period is over, comoving Hubble radius grow up along with the growth of universe. Based on the power spectrum obtained from the Starobinsky's model in the equation (25), at the cosmic inflation period, the rate of shrinking comoving Hubble radius is more rapidly than shrinking amplitude of scalar perturbation. Consequently, at the cosmic inflation epoch, there is possibility that amplitude of scalar perturbation on outside of comoving Hubble radius.

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