Preface

Soedirman's International Conference on Mathematics and Applied Sciences 2019

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October 23rd - 24th, 2019

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Soedirman's International Conference on Mathematics and Applied Sciences
SICoMAS 2019
23-24 October 2019, Purwokerto, Indonesia

Report from The Organizing Committee

It is indeed my great pleasure and honor to welcome you all to Soedirman's International Conference on Mathematics and Applied Sciences (SICoMAS) 2019. The conference running this year is the first SICoMAS series hosted by Faculty of Mathematics and Natural Sciences Jenderal Soedirman University. As the development of technology and management of world resources for our future based on the innovation in Mathematics and Sciences, this conference takes issue "Innovation in Mathematics and Applied Sciences for better future".

SICoMAS 2019 aims to provide a platform for researchers, lecturers, teachers, students, practitioners, and industrial professionals to share knowledge, exchange ideas, collaborate, and present research results in the fields of Mathematics, Chemistry, Physics, and their applications. Hence, my sincere gratitude goes to our four keynote speakers (Prof. Dr. Hadi Nur from University Teknologi Malaysia, Prof. Dr. Hirokazu Saito from Tokyo University of Science, Dr. Devi Putra , ST, M.Sc. from Pertamina Research and Tecnology, and Uyi Sulaeman, Ph.D. from Jenderal Soedirman University), and our six invited speakers (Prof. Dr. Youtoh Imai from Nishogakusha University, Prof. Riyanto, Ph.D. from Universitas Islam Indonesia, Dr. Moh. Adhib Ulil Absor from Gadjah Mada University, Bambang Hendriya Guswanto, Ph.D, Dadan Hermawan, Ph.D. and Dr. Eng. Mukhtar Effendi, M. Eng. from Jenderal Soedirman University) for sharing their expertise in this conference. My deepest appreciation also goes to our 80 presenters and 7 non presenters for their commitment to participate in this conference.

As the output of this conference, some selected papers in the field of chemistry will be published in Jurnal Molekul which is accredited Sinta 1; and other selected papers in the fields of Mathematics, Physics, Physical Chemistry, and Innovative Chemistry Education will be published in IOP Conference Series Journal. So, I greatly thank Jenderal Soedirman University, all our contributors, and all the members of the committee for the invaluable support that makes this conference a reality.
Finally, I would like to apologize for any short comings found in this conference; and hopefully this two-day conference will be engraved in your memory.

The chair of SICoMAS 2019

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All papers published in this volume of *Journal of Physics: Conference Series* have been peer reviewed through processes administered by the proceedings Editors. Reviews were conducted by expert referees to the professional and scientific standards expected of a proceedings journal published by IOP Publishing.

The validity of the properties of real numbers set to hyperreal numbers Set

The validity of the properties of real numbers set to hyperreal numbers Set

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Abstract. This research discusses the validity of the properties of real numbers set to hyperreal numbers set, i.e. algebraic, ordered, and completeness properties, by using a finitely additive measure. This finitely additive measure is a map from the power set of natural numbers set onto set $\{0,1\}$. The subset of natural numbers set has measure zero if it's finite and one if it's infinite. The set of hyperreal numbers is constructed from equivalence classes of the set of all sequences of real numbers by using a relation involving the finitely additive measure, that is, two sequences of real numbers are said to be related if and only if those two sequences are the same almost everywhere. In the hyperreal numbers set, there exist infinitesimal numbers besides 0. Infinitesimal number is a number which is less than any positive real number and greater than any negative real number. So, in hyperreal number set, there are some smallest positive numbers. The results show that the hyperreal numbers set is an ordered and complete field.

1. Introduction

The set $\mathbb{R}$ of real numbers can be constructed by using the concept of equivalence classes of the set of all rational Cauchy-sequence (see [3]). Abraham Robinson [2] had expanded the real numbers set. This expansion is called hyperreal numbers set ${}^*\mathbb{R}$. This numbers set is constructed by using the concept of equivalence classes of real numbers sequence. For example, both $\{1/n\}$ and $\{1/n^2\}$ are the sequences that converge to zero, but have different convergence rates. In ${}^*\mathbb{R}$, convergence rate is also considered. Then, in ${}^*\mathbb{R}$, both sequences are in different classes.

Another interesting property of hyperreal numbers set ${}^*\mathbb{R}$ is there exist more than one "small" or infinitesimal number. Infinitesimal number is a number which is less than any positive real numbers and greater than any negative real numbers. Differently from hyperreal numbers set ${}^*\mathbb{R}$, real numbers set $\mathbb{R}$ has one infinitesimal number, that is zero. Moreover, in ${}^*\mathbb{R}$, there is a "quantification" concept for "large" or infinite numbers. In ${}^*\mathbb{R}$, infinite property can be manifested as a number. Therefore, this paper investigates whether or not algebraic, ordered, and completeness properties, which are valid in real numbers set $\mathbb{R}$ (see [1]), are also valid in hyperreal numbers set ${}^*\mathbb{R}$.

2. Methods

The study of the validity of the properties of real numbers set $\mathbb{R}$ to hyperreal numbers set ${}^*\mathbb{R}$ is performanced by the following steps.

1. Constructing hyperreal numbers set using the concept of equivalence classes and measure $m: P(\mathbb{N}) \to \{0,1\}$ with $P(\mathbb{N})$ is the power set of natural numbers set $\mathbb{N}$.
2. Studying the properties of real numbers set which are valid in hyperreal numbers set, such as algebraic, ordered, and completeness properties by using the measure *m*.

3. Results and Discussion

Hyperreal numbers set ${}^*\mathbb{R}$ is constructed by using the concept of equivalence classes in the set of real numbers sequences (see [2]).

3.1 Construction of hyperreal numbers set ${}^*\mathbb{R}$

Hyperreal numbers set is constructed of using a measure $m: P(\mathbb{N}) \to \{0,1\}$ with $P(\mathbb{N})$ is the power set of natural numbers set $\mathbb{N}$ which is defined by the following definition.

Definition 3.1

The measure m is finitely additive measure on $P(\mathbb{N})$, if

i. for each $A \subseteq \mathbb{N}$, $m(A) \in \{0,1\}$;

ii. measure $m(\mathbb{N}) = 1$ *and* $m(A) = 0$ *for all finite set* $A \subseteq \mathbb{N}$.

The measure *m* is finitely additive measure which means $m(A \cup B) = m(A) + m(B)$, for all disjoint sets *A* and *B*. The measure *m* divides the subset of $\mathbb{N}$ into two parts, those are, a "large" or infinite set with a measure one and a "small" or finite set with a measure zero.

Any subset $A \subset \mathbb{N}$ satisfies one of the conditions $m(A) = 1$ or $m(A^c) = 1$. For any $A, B \subseteq \mathbb{N}$ with $m(A) = 1$ and $m(B) = 1$, then

$$m\left((A \cap B)^c\right) = m\left(A^c \cup B^c\right) \leq m\left(A^c\right) + m\left(B^c\right) = 0 + 0 = 0$$

which implies $m(A \cap B) = 1$. Next discussion is the construction of ${}^*\mathbb{R}$ by using the concept of equivalence classes on the set of all real numbers sequences.

Definition 3.2

Let $\Re$ *is the set of all real numbers sequences and* $\sim$ *is an equivalence relation on* $\Re$ *which is defined by the following. For all* $\{a_n\}, \{b_n\} \in \Re$

$$\{a_n\} \sim \{b_n\} \text{ if and only if } m\left(\{n : a_n = b_n\}\right) = 1. \quad (1)$$

In other words, $\{a_n\}$ is the same as $\{b_n\}$ almost everywhere. This following is the definition of hyperreal numbers set by using equivalence relation $\sim$.

Definition 3.3

Let $\{a_n\} \in \Re$. *The equivalence classes a with respect to the relation* $\sim$ *is defined by*

$${}^*a = \langle a_n \rangle = \{\{x_n\} \in \Re : \{x_n\} \sim \{a_n\}\}. \quad (2)$$

Next, hyperreal numbers set ${}^*\mathbb{R}$ *is a set of all equivalence classes in set* $\Re$ *and is denoted by*

$${}^*\mathbb{R} = \{\langle a_n \rangle : \{a_n\} \in \Re\}. \quad (3)$$

To study the properties of hyperreal numbers set which is analogous to real numbers set, the addition, multiplication, and order operation on ${}^*\mathbb{R}$ are defined.

Definition 3.4
For each $\langle a_n\rangle,\langle b_n\rangle \in {}^*\mathbb{R}$, *the addition operation "+", multiplication operation "·", and order operation "<" on* ${}^*\mathbb{R}$ *are defined by*

$$\langle a_n\rangle + \langle b_n\rangle = \langle a_n + b_n\rangle$$
$$\langle a_n\rangle \cdot \langle b_n\rangle = \langle a_n \cdot b_n\rangle$$
$$\langle a_n\rangle < \langle b_n\rangle \text{ if and only if } m\left(\{n : a_n < b_n\}\right) = 1. \quad (4)$$

Next, each $a \in \mathbb{R}$ can be written as ${}^*a = \langle a_n\rangle \in {}^*\mathbb{R}$ which means $m\left(\{n : a_n = a\}\right) = 1$. The set ${}^*\mathbb{R}$ is divided into three parts, those are infinitesimal numbers, finite numbers, and infinite numbers.

Definition 3.5
a) *An element* ${}^*x \in {}^*\mathbb{R}$ *is said to be infinitesimal if* $-a < {}^*x < a$ *for each* $a \in \mathbb{R}^+$.
b) *An element* ${}^*x \in {}^*\mathbb{R}$ *is said to be finite if* $-a < {}^*x < a$ *for some* $a \in \mathbb{R}^+$.
c) *An element* ${}^*x \in {}^*\mathbb{R}$ *is said to be infinite if* ${}^*x \le -a$ *or* ${}^*x \ge a$ *for each* $a \in \mathbb{R}^+$.

Example 3.1
The numbers ^{*0}, ${}^*\delta_1 = \langle 1/n\rangle$, and ${}^*\delta_2 = \langle 1/n^2\rangle$ in ${}^*\mathbb{R}$ are infinitesimal. Since $m\left(\{n : -a < 0_n < a\}\right) = 1$ then

$$m\left(\{n : -a < 1/n < a\}\right) = 1$$

and

$$m\left(\{n : -a < 1/n^2 < a\}\right) = 1$$

for each $a \in \mathbb{R}^+$, respectively. In addition, ${}^*\delta_1 = \langle 1/n\rangle$ and ${}^*\delta_2 = \langle 1/n^2\rangle$ are clearly two different numbers. The number $\langle n\rangle$ is infinite positive and number $\langle -n^2\rangle$ is infinite negative.

Theorem 3.1
Every ${}^*x \in {}^*\mathbb{R}$ *which is finite can be written uniquely as* ${}^*x = a + \varepsilon$ *with* $a \in \mathbb{R}$ *and* ε *is infinitesimal.*
Next, ${}^*x$ and ${}^*y$ are said to be infinitely close if ${}^*x - {}^*y$ infinitesimal. This relationship between ${}^*x$ and ${}^*y$ is denoted by ${}^*x \approx {}^*y$.

Definition 3.6
For each finite number ${}^*x \in {}^*\mathbb{R}$, *an unique real number a that satisfies* ${}^*x \approx a$ *is called the part of standard* ${}^*x$ *and denoted by* $\mathrm{st}({}^*x)$. *Conversely, for each* $a \in \mathbb{R}$, *the set of all* ${}^*x \in {}^*\mathbb{R}$ *with* $a = \mathrm{st}({}^*x)$ *is called monad a and denoted by* $\mathrm{mon}(a)$.

3.2 Internal set
A subset of ${}^*\mathbb{R}$ can be constructed by using measure *m* [2]. It is used to discusses the completeness properties of ${}^*\mathbb{R}$.

Definition 3.7
Let $A_n \in \mathbb{R}$ *where* $n = 1,\ 2,\ 3, \cdots$. *The sequence* $\{A_n\}$ *defines* $\langle A_n\rangle \subseteq {}^*\mathbb{R}$ *where*

$$\langle x_n \rangle \in \langle A_n \rangle \text{ if and only if } m(\{n : x_n \in A_n\}) = 1 \quad (5)$$

and the subset of ${}^*\mathbb{R}$ *which is obtained in this way is called an internal set.*

3.3 The properties of hyperreal numbers set

3.3.1 Algebraic properties. In hyperreal numbers set ${}^*\mathbb{R}$, there are two binary operations, those are hyperreal addition and hyperreal multiplication operation. Hyperreal numbers set ${}^*\mathbb{R}$ is a field with an identity element

$${}^*0 = \{\{0_n\} \in \mathfrak{R} : m(\{n : 0_n = 0\}) = 1\}$$

under the addition operation and the identity element

$${}^*1 = \{\{1_n\} \in \mathfrak{R} : m(\{n : 1_n = 1\}) = 1\}.$$

under the multiplication operation. Both identities under appropriate operations are unique. In addition, the multiplication of each element in ${}^*\mathbb{R}$ and ^{*0} is ^{*0}.

Theorem 3.2

a) If ${}^*z, {}^*a \in {}^*\mathbb{R}$ *and* ${}^*z + {}^*a = {}^*a$, *then* ${}^*z = {}^*0$

b) If ${}^*u, {}^*b \in {}^*\mathbb{R}$, ${}^*b \neq {}^*0$, *and* ${}^*u \cdot {}^*b = {}^*b$, *then* ${}^*u = {}^*1$

c) If ${}^*a \in {}^*\mathbb{R}$, *then* ${}^*a \cdot {}^*0 = {}^*0$

Prove 3.2.a)

Let ${}^*z, {}^*a \in {}^*\mathbb{R}$ where ${}^*z = \langle z_n \rangle$ and ${}^*a = \langle a_n \rangle$. Observe that ${}^*z + {}^*a = {}^*a$ which means $\langle z_n \rangle + \langle a_n \rangle = \langle a_n \rangle$ and based on addition definition in ${}^*\mathbb{R}$, $\langle z_n + a_n \rangle = \langle a_n \rangle$ or in other words $m(\{n : z_n + a_n = a_n\}) = 1$. For each $n \in \{n : z_n + a_n = a_n\}$, if $z_n + a_n = a_n$ then $z_n = 0$. Since $m(\{n : z_n + a_n = a_n\}) = 1$ then $m(\{n : z_n = 0\}) = 1$. So, it can be proven that ${}^*z = {}^*0$. ■

Furthermore, under multiplication operation, every element in ${}^*\mathbb{R} \setminus \{{}^*0\}$ has a unique inverse element and the multiplication of two numbers in ${}^*\mathbb{R}$ that produces ^{*0} is fulfilled when one or both of these numbers is ^{*0}.

Theorem 3.3

a) If ${}^*a \neq {}^*0$ *and* ${}^*a, {}^*b \in {}^*\mathbb{R}$ *such that* ${}^*a \cdot {}^*b = {}^*1$, *then* ${}^*b = 1/{}^*a$.

b) If ${}^*a, {}^*b \in {}^*\mathbb{R}$ *and* ${}^*a \cdot {}^*b = {}^*0$ *then* ${}^*a = {}^*0$ *or* ${}^*b = {}^*0$.

Prove 3.3.b)

Let ${}^*a, {}^*b \in {}^*\mathbb{R}$ where ${}^*a = \langle a_n \rangle$ and ${}^*b = \langle b_n \rangle$. Observe that

$$\langle a_n \rangle \cdot \langle b_n \rangle = \langle a_n \cdot b_n \rangle,$$

which means

$$m(\{n : a_n \cdot b_n = 0\}) = 1.$$

Also, observe that

$$\{n : a_n \cdot b_n = 0\} = \{n : a_n = 0\} \cup \{n : b_n = 0\}.$$

If $m(\{n:a_n=0\})=0$ and $m(\{n:b_n=0\})=0$, then $\{n:a_n=0\}$ and $\{n:b_n=0\}$ are finite sets. Consequently, $\{n:a_n\cdot b_n=0\}$ is finite or $m(\{n:a_n\cdot b_n=0\})=0$. In this case, there is a contradiction. So, it must be $m(\{n:a_n=0\})=1$ or $m(\{n:b_n=0\})=1$. In other words, ${}^*a={}^*0$ or ${}^*b={}^*0$ ■

3.3.2 Ordered Properties. Ordered properties in hyperreal numbers set is analogous to the ordered properties in real numbers set.

Theorem 3.4

a) If ${}^*a, {}^*b, {}^*c\in{}^*\mathbb{R}$ *such that* ${}^*a>{}^*b$ *and* ${}^*b>{}^*c$ *then* ${}^*a>{}^*c$*;*

b) If ${}^*a, {}^*b, {}^*c\in{}^*\mathbb{R}$ *and* ${}^*a>{}^*b$ *then* ${}^*a+{}^*c>{}^*b+{}^*c$*;*

c) If ${}^*a, {}^*b, {}^*c\in{}^*\mathbb{R}$ *such that* ${}^*a>{}^*0$ *and* ${}^*b<{}^*c$*, then* ${}^*a\,{}^*b<{}^*a\,{}^*c$*, also if* ${}^*a, {}^*b, {}^*c\in{}^*\mathbb{R}$ *such that* ${}^*a<{}^*0$ *and* ${}^*b<{}^*c$ *then* ${}^*a\,{}^*b>{}^*a\,{}^*c$*;*

d) If ${}^*a, {}^*b\in{}^*\mathbb{R}$ *and* ${}^*0=\langle 0_n\rangle\in{}^*\mathbb{R}$ *such that* ${}^*a\,{}^*b>{}^*0$*, then* ${}^*a>{}^*0$ *and* ${}^*b>{}^*0$*, or* ${}^*a<{}^*0$ *and* ${}^*b<{}^*0$*;*

e) *If* ${}^*a, {}^*b\in{}^*\mathbb{R}$ *and* ${}^*0=\langle 0_n\rangle\in{}^*\mathbb{R}$ *such that* ${}^*a\,{}^*b<{}^*0$*, then* ${}^*a>{}^*0$ *and* ${}^*b<{}^*0$*, or* ${}^*a<{}^*0$ *and* ${}^*b>{}^*0$.

Prove

a) We must prove that ${}^*a>{}^*c$ with ${}^*a=\langle a_n\rangle$, ${}^*b=\langle b_n\rangle$, and ${}^*c=\langle c_n\rangle$. Observe that ${}^*a>{}^*b$ and ${}^*b>{}^*c$ which mean $\langle a_n\rangle>\langle b_n\rangle$ and $\langle b_n\rangle>\langle c_n\rangle$. This means that $m(\{n:a_n>b_n\})=1$ and $m(\{n:b_n>c_n\})=1$. We must show that $m(\{n:a_n>c_n\})=1$. For every $n\in\{n:a_n>b_n\}\cap\{n:b_n>c_n\}$, if $a_n>b_n$ and $b_n>c_n$ then $a_n>c_n$. Next, since $m(\{n:a_n>b_n\})=1$ and $m(\{n:b_n>c_n\})=1$, then

$$m(\{n:a_n>b_n\}\cap\{n:b_n>c_n\})=1.$$

Consequently, $m(\{n:a_n>c_n\})=1$. So, it can be proven that ${}^*a>{}^*c$.

b) We must prove that ${}^*a+{}^*c>{}^*b+{}^*c$ with ${}^*a=\langle a_n\rangle$, ${}^*b=\langle b_n\rangle$, and ${}^*c=\langle c_n\rangle$. Observe that ${}^*a>{}^*b$ or $\langle a_n\rangle>\langle b_n\rangle$. This means $m(\{n:a_n>b_n\})=1$. We must show that $m(\{n:a_n+c_n>b_n+c_n\})=1$. For every $n\in\{n:a_n>b_n\}$, if $a_n>b_n$ then $a_n+c_n>b_n+c_n$. Next, since $m(\{n:a_n>b_n\})=1$. then

$$m(\{n:a_n+c_n>b_n+c_n\})=1.$$

So, we can prove that ${}^*a+{}^*c>{}^*b+{}^*c$.

c) We must prove that ${}^*a\,{}^*b<{}^*a\,{}^*c$ with ${}^*a=\langle a_n\rangle$, ${}^*b=\langle b_n\rangle$, and ${}^*c=\langle c_n\rangle$. Observe that ${}^*a>{}^*0$ and ${}^*b<{}^*c$ which mean $\langle a_n\rangle>\langle 0_n\rangle$ and $\langle b_n\rangle<\langle c_n\rangle$. This means $m(\{n:a_n>0\})=1$ and $m(\{n:b_n<c_n\})=1$. We must show that $m(\{n:a_nb_n<a_nc_n\})=1$. For every $n\in\{n:a_n>0\}\cap\{n:b_n<c_n\}$, if $a_n>0$ and $b_n<c_n$ then $a_nb_n<a_nc_n$. Next, since

$$m(\{n:a_n>0\})=1$$

and

$$m(\{n:b_n<c_n\})=1$$

then $m(\{n:a_n>0\}\cap\{n:b_n<c_n\})=1$. Therefore, $m(\{n:a_nb_n<a_nc_n\})=1$. So, we proved that ${}^*a\,{}^*b<{}^*a\,{}^*c$. Using the same way, it can be proven that if ${}^*a, {}^*b, {}^*c\in{}^*\mathbb{R}$ such that ${}^*a<{}^*0$ and ${}^*b<{}^*c$, then ${}^*a\,{}^*b>{}^*a\,{}^*c$.

d) We must prove that if ${}^*a\,{}^*b>{}^*0$ then ${}^*a>{}^*0$ and ${}^*b>{}^*0$, or ${}^*a<{}^*0$ and ${}^*b<{}^*0$ with ${}^*a=\langle a_n\rangle$, ${}^*b=\langle b_n\rangle$, and ${}^*0=\langle 0_n\rangle\in{}^*\mathbb{R}$. In order to do that, We must show that $\langle a_n\rangle>\langle 0_n\rangle$ and $\langle b_n\rangle>\langle 0_n\rangle$, or $\langle a_n\rangle<\langle 0_n\rangle$ and $\langle b_n\rangle<\langle 0_n\rangle$. In other words, we must show that $m(\{n:a_n>0\})=1$ and $m(\{n:b_n>0\})=1$, or $m(\{n:a_n<0\})=1$ and $m(\{n:b_n<0\})=1$. Observe that ${}^*a\,{}^*b>{}^*0$ which means $m(\{n:a_nb_n>0\})=1$ and

$$\{n:a_nb_n>0\}=\{n:a_n>0 \text{ and } b_n>0\}\cup\{n:a_n<0 \text{ and } b_n<0\}.$$

Since $m(\{n:a_nb_n>0\})=1$, then

$$m(\{n:a_n>0 \text{ and } b_n>0\}\cup\{n:a_n<0 \text{ and } b_n<0\})=1.$$

Suppose

$$m(\{n:a_n>0 \text{ and } b_n>0\})=0$$

and

$$m(\{n:a_n<0 \text{ and } b_n<0\})=0.$$

Consequently,

$$m(\{n:a_n>0 \text{ and } b_n>0\}\cup\{n:a_n<0 \text{ and } b_n<0\})=m(\{n:a_n>0 \text{ and } b_n>0\})+m(\{n:a_n<0 \text{ and } b_n<0\})$$

$$=0+0$$
$$=0.$$

This contradicts to

$$m(\{n:a_nb_n>0\})=m(\{n:a_n>0 \text{ and } b_n>0\}\cup\{n:a_n<0 \text{ and } b_n<0\})=1.$$

So, must

$$m(\{n:a_n>0 \text{ and } b_n>0\})=1$$

or

$$m(\{n:a_n<0 \text{ and } b_n<0\})=1.$$

e) Using the same way as in proof of d), we can prove that if ${}^*a\,{}^*b<{}^*0$, then ${}^*a>{}^*0$ and ${}^*b<{}^*0$, or ${}^*a<{}^*0$ and ${}^*b>{}^*0$. ■

In real numbers set, natural numbers are positive real numbers[1]. Also, natural numbers in hyperreal numbers set are positive hyperreal numbers. Moreover, in real numbers set, there is no smallest positive real numbers[1]. In hyperreal numbers set, there exist infinitesimal numbers, for example $\langle 1/n\rangle$, since $m(\{n:-a<1/n<a\})=1$ for each $a\in\mathbb{R}^+$. Therefore, in hyperreal numbers set, there are smallest positive hyperreal numbers.

3.3.3 Completeness Properties. This following is a discussion of completeness properties of ${}^*\mathbb{R}$, that is the completeness properties which is related to the supremum and infimum concepts in ${}^*\mathbb{R}$.

Definition 3.8

Let ${}^*A$ is a non-empty subset of ${}^*\mathbb{R}$.

a) The set ${}^*A$ is called bounded above if there exists ${}^*u \in {}^*\mathbb{R}$ such that ${}^*a \leq {}^*u$ for each ${}^*a \in {}^*A$. The hyperreal number such ${}^*u$ is called as upper bound of ${}^*A$.

b) The set ${}^*A$ is called bounded below if there exists ${}^*v \in {}^*\mathbb{R}$ such that ${}^*v \leq {}^*a$ for each ${}^*a \in {}^*A$. The hyperreal number such ${}^*v$ is called lower bound of ${}^*A$.

c) The set ${}^*A$ is called bounded if ${}^*A$ is both bounded above and bounded below. Conversely, a set ${}^*A$ is called unbounded, if ${}^*A$ is not bounded above or not bounded below.

Definition 3.9

Let ${}^*A$ is a non-empty subset of ${}^*\mathbb{R}$.

a) If ${}^*A$ bounded above, then a number ${}^*u$ is called a supremum (least upper bound) of ${}^*A$ if it satisfies these conditions:

i. ${}^*u$ is an upper bound of ${}^*A$;

ii. if ${}^*s$ is any upper bound of ${}^*A$, then ${}^*u \leq {}^*s$.

b) If ${}^*A$ bounded below, then ${}^*v$ is called an infimum (greatest lower bound) of ${}^*A$ if it satisfies these conditions:

i. ${}^*v$ is a lower bound of ${}^*A$;

ii. if ${}^*t$ is any lower bound of ${}^*A$, then ${}^*t \leq {}^*v$.

Theorem 3.5

Let ${}^*u$, ${}^*v \in {}^*\mathbb{R}$, and ${}^*A$ is a non-empty subset of ${}^*\mathbb{R}$.

a) An element ${}^*u$ is a supremum ${}^*A$ if and only if ${}^*z < {}^*u$ implies ${}^*a^z > {}^*z$ for some ${}^*a^z \in {}^*A$.

b) An element ${}^*v$ is an infimum ${}^*A$ if and only if ${}^*z > {}^*v$ implies ${}^*a^z < {}^*z$ for some ${}^*a^z \in {}^*A$.

Prove 3.5.a)

Since ${}^*A = \langle A_i \rangle$ is a non-empty internal set of ${}^*\mathbb{R}$, then there exist ${}^*a \in {}^*A$ such that $m(\{i : a_i \in A_i\}) = 1$. Let ${}^*u = \sup {}^*A$ is supremum ${}^*A$, which means for every ${}^*a \in {}^*A$, $m(\{i : a_i \leq u_i = \sup A_i\}) = 1$. Based on supremum theorem in real numbers set, u_i is a supremum A_i if and only if, for each $i \in \{i : a_i \leq u_i\}$, if $z_i < u_i$ then there exists $a_i^z \in A_i$ such that $z_i < a_i^z$ [1]. Since $m(\{i : a_i \leq u_i\}) = 1$, then

$$m\left(\left\{i : z_i < a_i^z\right\}\right) = 1.$$

Consequently, if ${}^*z = \langle z_i \rangle < {}^*u$ then there exists ${}^*a^z = \langle a_i^z \rangle$ such that ${}^*z < {}^*a^z$. ■

Theorem 3.6

Let ${}^*u$, ${}^*v \in {}^*\mathbb{R}$, ${}^*A$ is a non-empty internal set of ${}^*\mathbb{R}$.

a) An element ${}^*u$ is a supremum of ${}^*A$, if and only if for every ${}^*\varepsilon > {}^*0$ there exists ${}^*a^\varepsilon \in {}^*A$ such that ${}^*a^\varepsilon > {}^*u - {}^*\varepsilon$.

b) An element ${}^*v$ is an infimum of ${}^*A$, if and only if for each ${}^*\varepsilon > {}^*0$ there exist ${}^*a^\varepsilon \in {}^*A$ such that ${}^*a^\varepsilon < {}^*v + {}^*\varepsilon$.

Prove 3.6.a)

Since ${}^*A = \langle A_i \rangle$ is a non-empty internal set of ${}^*\mathbb{R}$, then there exist ${}^*a \in {}^*A$ such that $m(\{i : a_i \in A_i\}) = 1$. Let ${}^*u = \sup {}^*A$ is a supremum of ${}^*A$, which means for each ${}^*a \in {}^*A$, $m(\{i : a_i \leq u_i = \sup A_i\}) = 1$. Based on supremum theorem of real numbers set, u_i is a supremum A_i if and only if, for each $i \in \{i : a_i \leq u_i\}$, every $\varepsilon_i > 0$, there exist $a_i^\varepsilon \in A_i$ such that $a_i^\varepsilon > u_i - \varepsilon_i$ [1]. Since $m(\{i : a_i \leq u_i\}) = 1$, then

$m\left(\left\{i : a_i^{\varepsilon} > u_i - \varepsilon_i\right\}\right) = 1$. Consequently, for each ${}^*\varepsilon = \langle \varepsilon_i \rangle > {}^*0$ there exists ${}^*a^{\varepsilon} = \left\langle a_i^{\varepsilon} \right\rangle \in {}^*A$ such that ${}^*a^{\varepsilon} > {}^*u - {}^*\varepsilon$. ■

Completeness Properties

a) Every non-empty internal set of ${}^*\mathbb{R}$ that has an upper bound also has a least upper bound.
b) Every non-empty internal set of ${}^*\mathbb{R}$ that has a lower bound also has a greatest lower bound.

Prove a)

Let ${}^*A = \langle A_i \rangle$ is an internal set in ${}^*\mathbb{R}$ which is bounded above ${}^*a = \langle a_i \rangle$. For each ${}^*x \in {}^*A$, ${}^*x \leq {}^*a$ which means if $\langle x_i \rangle \in \langle A_i \rangle$ then $\langle x_i \rangle \leq \langle a_i \rangle$. This means $m\left(\left\{i : x_i \in A_i\right\}\right) = 1$ and $m\left(\left\{i : x_i \leq a_i\right\}\right) = 1$. consequently, almost every A_i bounded above by a_i. Next, we must prove that ${}^*b = \langle \sup A_i \rangle$ is a sup ${}^*A$ by showing:

i. for each $\langle x_i \rangle \in \langle A_i \rangle$, $m\left(\left\{i : \sup A_i \geq x_i\right\}\right) = 1$.

ii. for each upper bound ${}^*a$ of ${}^*A$, $m\left(\left\{i : \sup A_i \leq a_i\right\}\right) = 1$.

Since $\sup A_i$ is a least upper bound of A_i, then for every ${}^*x \in {}^*A$, $m\left(\left\{i : \sup A_i \geq x_i\right\}\right) = 1$ and for every upper bound ${}^*a$ of ${}^*A$, $m\left(\left\{i : \sup A_i \leq a_i\right\}\right) = 1$. So, it can be proven that ${}^*b = \langle \sup A_i \rangle$ is least upper bound of ${}^*A$. ■

Hyperreal numbers set can be described as follows.

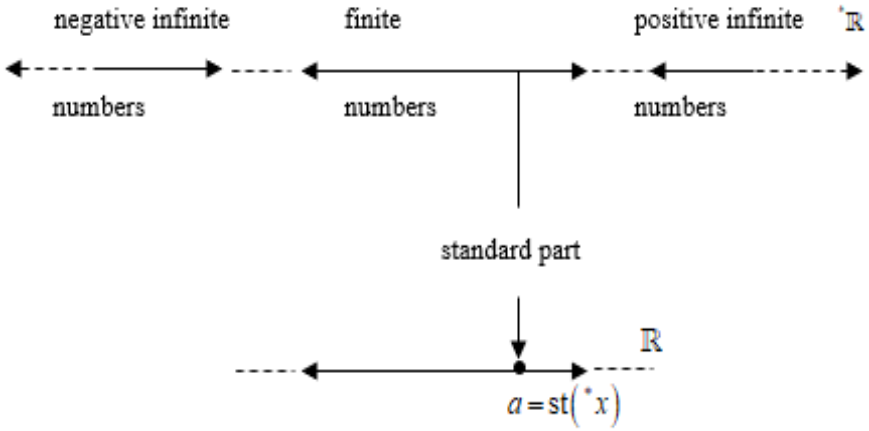

Figure 3.1 Hyperreal Numbers Set

Based on Figure 3.1, hyperreal numbers set is divided into three parts, those are negative infinite, finite, and positive infinite numbers. Based on Theorem 3.1, standard part of hyperreal numbers which are finite, are real numbers. In other words, the set of all standard part of hyperreal numbers which is finite is real numbers set.

4. Conclusions

Based on result and discussion above, it can be concluded that hyperreal numbers set ${}^*\mathbb{R}$ is the set of all equivalence classes in set $\Re$ and denoted by

$${}^*\mathbb{R} = \{\langle a_n \rangle : \{a_n\} \in \Re\}.$$

Hyperreal numbers set is a field with the identity element under the addition operation, that is

$$^{*}0=\left\{\left\{0_n\right\}\in\Re : m\left(\left\{n:0_n=0\right\}\right)=1\right\}$$

and the identity element under the multiplication operation, that is

$$^{*}1=\left\{\left\{1_n\right\}\in\Re : m\left(\left\{n:1_n=1\right\}\right)=1\right\}.$$

The ordered properties of real numbers set also valid to hyperreal numbers set, except the ordered properties of $\mathbb{R}$ which says there is no a smallest positive real number. In the hyperreal numbers set there are infinitesimal numbers other than zero, for example $\langle 1/n\rangle$. All completeness properties in real numbers set is also valid in hyperreal numbers set $^{*}\mathbb{R}$. Thus, it can be concluded that the hyperreal numbers set is an ordered and complete field.

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